1. Let m be a positive integer, and a be an integer such that $\gcd(a, m) = 1$. We showed in class that there must exist some exponent $e > 0$ such that $a^e \equiv 1 \pmod{m}$. Call the smallest such value e the *multiplicative order* of a modulo m.

   (a) For each a between 1 and 12 inclusive, find the multiplicative order of a modulo 13.

   (b) For each a between 1 and 14 inclusive such that $\gcd(a, 15) = 1$, find the multiplicative order of a modulo 15.

2. Let a and b be two positive integers such that $\gcd(a, b) = 1$. Suppose that x, y are two other integers such that $x \equiv y \pmod{a}$ and $x \equiv y \pmod{b}$. Prove that $x \equiv y \pmod{ab}$.

3. Make a 6×7 grid, where the rows are labelled 0 through 5 (inclusive) and the columns are labelled 0 through 6 (inclusive). In the cell of the grid in row r and column c, write a number x between 0 and 41 inclusive such that $x \equiv r \pmod{6}$ and $x \equiv c \pmod{7}$ (the previous problem shows that there is exactly one choice for each cell).

   Hint. You may find it easier to place the numbers $0, 1, 2, 3, \cdots, 41$ in that order, rather than filling out the chart one row at a time or one column at a time.

4. Compute $\phi(97)$ and $\phi(8800)$, where $\phi(n)$ denotes Euler's phi function.

5. Determine the last two digits (tens digit and units digit) of 19^{5085}.