1. How many ways are there to pay 79 dollars using a combination of five dollar bills and two dollar bills (without receiving any change)?

2. (a) Find integers $w, z$ such that $3w + 20z = 1$.
(b) Let $w$ be from your answer to the previous part. Find integers $x$ and $y$ such that $6x + 15y = 3w$.
(c) Put together your previous two answers to give a solution to the equation $6x + 15y + 20z = 1$, with $x, y, z$ integers.
(d) Follow a similar method to find an integer solution to the equation $155x + 341y + 385z = 1$.

3. Suppose that $a, b$ are two natural numbers such that $\gcd(a, b) = 1$. Suppose that $c$ is a natural number which is divisible by both $a$ and $b$. Show that $c$ is divisible by the product $ab$.

4. The *Fibonacci numbers* are defined as follows: $F_0 = 0$, $F_1 = 1$, and for all $n > 1$, 
\[ F_n = F_{n-1} + F_{n-2}. \]
For example, the next several Fibonacci numbers are $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$.
(a) For each value of $n$ from 3 to 9, find integers $x_n$ and $y_n$ such that $x_n F_n + y_n F_{n+1} = 1$.
Arrange these values into a table (you may notice a pattern in this table).
(b) Prove by induction on $n$ that for all $n \geq 0$, $\gcd(F_n, F_{n+1}) = 1$.

5. Find all incongruent solutions $x$ to each of the following congruences.
(a) $7x \equiv 4 \pmod{5}$
(b) $x^2 \equiv 3 \pmod{13}$
(c) $9x \equiv 6 \pmod{15}$

Due Friday 2/13 in class.