1. How many ways are there to pay 79 dollars using a combination of five dollar bills and two dollar bills (without receiving any change)?

2. (a) Find integers w, z such that $3w + 20z = 1$.
   (b) Let w be from your answer to the previous part. Find integers x and y such that $6x + 15y = 3w$.
   (c) Put together your previous two answers to give a solution to the equation $6x+15y+20z = 1$, with x, y, z integers.
   (d) Follow a similar method to find an integer solution to the equation $155x+341y+385z = 1$.

3. Suppose that a, b are two natural numbers such that $\gcd(a, b) = 1$. Suppose that c is a natural number which is divisible by both a and b. Show that c is divisible by the product ab.

4. The *Fibonacci numbers* are defined as follows: $F_0 = 0,\ F_1 = 1$, and for all $n > 1$,

$$F_n = F_{n-1} + F_{n-2}.$$

   For example, the next several Fibonacci numbers are $F_2 = 1,\ F_3 = 2,\ F_4 = 3,\ F_5 = 5,\ F_6 = 8$.

   (a) For each value of n from 3 to 9, find integers x_n and y_n such that

$$x_n F_n + y_n F_{n+1} = 1.$$

   Arrange these values into a table (you may notice a pattern in this table).

   (b) Prove by induction on n that for all $n \geq 0$, $\gcd(F_n, F_{n+1}) = 1$.

5. Find all incongruent solutions x to each of the following congruences.
   (a) $7x \equiv 4 \pmod{5}$
   (b) $x^2 \equiv 3 \pmod{13}$
   (c) $9x \equiv 6 \pmod{15}$