

1. Fill out the (short) course survey here. You do not need to submit anything about this with the assignment.

<http://goo.gl/forms/0MnAuJx4g1>

2. *Fermat's Last Theorem* states that for any natural number $n \geq 3$, the equation $a^n + b^n = c^n$ has no solutions for natural numbers a, b, c . Prove that if Fermat's last theorem is true for the special case $n = 4$, and also whenever n is equal to an odd prime number, then Fermat's last theorem is true for all $n \geq 3$.
3. (a) Follow the method of chapter 3 to describe all the *rational* solutions to the equation $x^2 + y^2 = 2$. Note that you will need to choose a different "center of projection" than was used for the equation $x^2 + y^2 = 1$.
(b) Using your answer to part (a), find an *natural number* (i.e. positive integer) solution to the equation $a^2 + b^2 = 2c^2$, such that $a \neq b$.

Note. For the following two problems, please compute the answer *by hand*, showing all of your steps.

4. Using the Euclidean algorithm, compute each of the following.

(a) $\gcd(180, 364)$

(b) $\gcd(1001, 1456)$

5. Use the Euclidean algorithm to find an *integer* solution to the following equation

$$181x + 293y = 1$$

6. (a) Suppose that you have a large supply of 5 dollar coins and a large supply of 6 dollar coins. What is the largest number of dollars that you can *not* make out of some combination of these coins? You do not need to prove your answer, but briefly describe how you have decided upon it.
(b) Suppose instead that you have 5 dollar and 7 dollar coins. What is the largest amount that you cannot make?
(c) Suppose instead that you have 5 dollar coins and 8 dollar coins. What is the largest amount that you cannot make?
(d) Conjecture a formula for the largest amount that you cannot make out of 5 dollar coins and k dollar coins, where k is any integer greater than 1 that is not divisible by 5. You do not need to prove that your formula is correct (but you are encouraged to attempt to do so!).