If you wish to complete the optional programming assignment, submissions for the first task (called "Dirichlet") are also due by 2pm on Friday 10 April.

1. For each congruence, determine whether or not a solution exists. You do not need to find the solution. (*Note.* Each modulus is prime.)

   (a) $x^2 \equiv -1 \pmod{5987}$

   (b) $x^2 \equiv 6780 \pmod{6781}$

   (c) $x^2 + 14x - 35 \equiv 0 \pmod{337}$

   (d) $x^2 - 64x + 943 \equiv 0 \pmod{3011}$

2. Evaluate each of the following Legendre symbols.

   (a) $\left(\frac{85}{101}\right)$

   (b) $\left(\frac{29}{541}\right)$

   (c) $\left(\frac{101}{1987}\right)$

   (d) $\left(\frac{31706}{43789}\right)$

3. Let n be an number such that $n + 5$ is a perfect square. Prove that every prime factor of n, besides possibly 2 and possibly 5, is congruent to 1 or 4 modulo 5.

4. Suppose that p be a prime number besides 2 or 3. Show that the value of the Legendre symbol $\left(\frac{3}{p}\right)$ only depends on the remainder when p is divided by 12. Which remainders correspond to primes p such that 3 is a quadratic residue?