Worksheet for 11/18/13

Review: give some antiderivative for each function.

a) x^{-1} $\qquad \ln|x|$

b) 1 $\qquad x$

c) x $\qquad \frac{1}{2}x^2$

d) x^2 $\qquad \frac{1}{3}x^3$

e) $\sqrt{x}$ $\qquad \frac{2}{3}x^{3/2}$

f) $\frac{1}{\sqrt{x}}$ $\qquad 2\sqrt{x}$

g) e^x $\qquad e^x$

h) 2^x $\qquad \frac{1}{\ln 2}\cdot 2^x$

i) $\cos x$ $\qquad \sin x$

j) $\sin x$ $\qquad -\cos x$

k) $\sec^2 x$ $\qquad \tan x$

l) $\sec x \cdot \tan x$ $\qquad \sec x$

m) $\frac{1}{\sqrt{1-x^2}}$ $\qquad \sin^{-1}x$

n) $\frac{1}{1+x^2}$ $\qquad \tan^{-1}x$

Review these enough that you can produce them almost instantly.

Evaluate:

* 1) $\int_1^4 3\sqrt{x}\,dx = \left[\cancel{3}\cdot\frac{2}{\cancel{3}}\cdot x^{3/2}\right]_1^4 = 2\cdot 2^{3/2} - 2\cdot 1^{3/2}$

$= 16-2 = \boxed{14}$

* 2) $\int_0^{\pi} 5\sin x\,dx = \left[-5\cos x\right]_0^{\pi} = 5-(-5) = \boxed{10}$

* 3) $\int_x^{7x} \frac{dt}{t}$ (in terms of x) $= \left[\ln|t|\right]_x^{7x}$

$= \ln|7x| - \ln|x| = \boxed{\ln 7}$

(4) $\int_0^{10} e^{-t}dt = [-e^{-t}]_0^{10}$

$= -e^{-10} + e^0$

$= \boxed{1-e^{-10}}$

(5) $\int_h^1 \frac{1}{\sqrt{x}}\,dx$ (in terms of h)

$= [2\sqrt{x}]_h^1$

$= \boxed{2-2\sqrt{h}}$

(6) $\int_{-\pi/3}^{\pi/3} (2-\sec^2\theta)\,d\theta$

$= [2\theta - \tan\theta]_{-\pi/3}^{\pi/3}$

$= \left(\frac{2\pi}{3} - \sqrt{3}\right) - \left(-\frac{2\pi}{3} + \sqrt{3}\right) = \boxed{\frac{4\pi}{3} - 2\sqrt{3}}$

(7) $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

$= [\tan^{-1}x]_{-1}^{1}$

$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \boxed{\frac{\pi}{2}}$

(8) $\int_{-1/2}^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx$

$= [\sin^{-1}x]_{-1/2}^{1/2} = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \boxed{\frac{\pi}{3}}$

Part II

① $\int e^{-2x}dx = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C = \boxed{-\frac{1}{2}e^{-2x} + C}$

$u = -2x$
$du = -2dx$

② $\int \frac{1}{\sqrt{x+7}}dx = \int \frac{1}{\sqrt{u}}du = \boxed{2\sqrt{x+7} + C}$

$u = x+7$
$\cancel{du} = dx$

* ③ $\int \cos(5x+7)dx = \frac{1}{5}\sin u + C = \boxed{\frac{1}{5}\sin(5x+7) + C}$

$u = 5x+7$
$du = 5dx$

④ $\int x^2\sqrt{x^3+1}\,dx = \int \frac{1}{3}\sqrt{u}\cdot du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{9}(x^3+1)^{3/2} + C}$

$u = x^3+1$
$du = 3x^2dx$

* ⑤ $\int x\cdot e^{x^2}dx = \int \frac{1}{2}e^u du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{x^2} + C}$

$u = x^2$
$du = 2xdx$

⑥ $\int 3x\sqrt{x^2+1}\,dx = \int \frac{3}{2}\sqrt{u}\,du = u^{3/2} + C = \boxed{(x^2+1)^{3/2} + C}$

$u = x^2+1$
$du = 2x\,dx$

⑦ $\int \frac{\cos x}{\sin^2 x}dx = \int \frac{du}{u^2} = -\frac{1}{u} + C = \boxed{-\frac{1}{\sin x} + C}$ or $\underline{-\csc x + C}$

(same thing)

$u = \sin x$
$du = \cos x\,dx$

* ⑧ $\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx = \int \frac{-du}{u} = -\ln|u| + C$

$= \boxed{-\ln|\cos x| + C}$

or $\underline{\ln|\sec x| + C.}$

(same thing)

$u = \cos x$
$du = -\sin x\,dx$

* (9) $\int e^x \cos(e^x)\,dx = \int \cos(u)\,du$

$u = e^x$

$du = e^x dx$

$= \sin u + C$

$= \boxed{\sin(e^x) + C}$

(10) $\int x\sqrt{x+1}\,dx = \int (u-1)\sqrt{u}\,du = \int (u^{3/2} - \sqrt{u})\,du$

$u = x+1$

$du = dx$

$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$

Challenges (some will be discussed on Thursday)

(11) $\int \cos^3 x \sin^2 x\,dx = \int \cos^2 x \cdot u^2\,du = \int (1-u^2)\cdot u^2\,du$

$u = \sin x$

$du = \cos x\,dx$

$= \int (u^2 - u^4)\,du = \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

(12) $\int \frac{e^{2x}}{e^x+1}\,dx = \int \frac{e^x du}{u} = \int \frac{(u-1)}{u}\,du = \int \left(1 - \frac{1}{u}\right) du$

$u = e^x + 1$

$du = e^x dx$

$= u - \ln|u| + C = \boxed{e^x + 1 - \ln(e^x+1) + C}$

or just $e^x - \ln(e^x+1) + C$.

(subsume the 1 into the C)

(13) $\int \tan^4 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

use: $\sec^2 x - 1 = \tan^2 x$.

$= \int (\sec^2 x - 1)\tan^2 x\,dx$

$= \int (\sec^2 x \tan^2 x - \tan^2 x)\,dx$

$= \int (\sec^2 x \tan^2 x - \sec^2 x + 1)\,dx$

$= \int (\tan^2 x - 1)\sec^2 x\,dx + \int 1\,dx$

$= \int (u^2 - 1)\,du + x + C$

$= \frac{1}{3}u^3 - u + x + C$

$= \boxed{\frac{1}{3}\tan^3 x - \tan x + x + C}$