Math 19 Review Problems 12/8/14

① Evaluate $\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx = \int_0^{\pi/2} \sin x \cdot (1-\cos^2 x)\cos^2 x \, dx$

$u = \cos x \quad du = -\sin x\, dx$

$= \int_1^0 (1-u^2)\cdot u^2 \cdot (-1)\, du = \int_0^1 (u^2 - u^4)\, du$

$= \left[\frac{1}{3}u^3 - \frac{1}{5}u^5\right]_0^1 = \frac{1}{3} - \frac{1}{5} = \boxed{2/15}$

② Evaluate $\int \sqrt{16-x^2}\, dx$

$x = 4\sin\theta \qquad dx = 4\cos\theta\, d\theta$

$\sqrt{16-x^2} = 4\cos\theta$

$= \int 16\cos^2\theta\, d\theta = \int 8(1+\cos(2\theta))\, d\theta = 8\theta + 4\sin(2\theta) + C$

$= \boxed{8\cdot \sin^{-1}\left(\frac{x}{4}\right) + 4\cdot \sin\left(2\cdot\sin^{-1}\left(\frac{x}{4}\right)\right) + C}$

③ Find the side lengths & angles of the triangle w/ vertices at:

$\vec{A} = (1,2,0)$

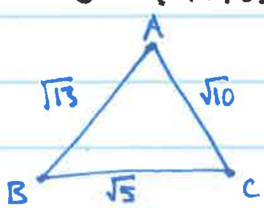
$\vec{B} = (1,0,3)$

$\vec{C} = (0,2,3)$

$|\overrightarrow{AB}| = |(1,0,3)-(1,2,0)| = |(0,-2,3)| = \sqrt{13}$

$|\overrightarrow{BC}| = |(0,2,3)-(1,0,3)| = |(-1,2,0)| = \sqrt{5}$

$|\overrightarrow{CA}| = |(1,2,0)-(0,2,3)| = |(1,0,-3)| = \sqrt{10}$



$\angle A = \cos^{-1}\left(\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\|\overrightarrow{AB}\|\cdot\|\overrightarrow{AC}\|}\right) = \cos^{-1}\left(\frac{9}{\sqrt{130}}\right)$

$\angle B = \cos^{-1}\left(\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{\|\overrightarrow{BA}\|\cdot\|\overrightarrow{BC}\|}\right) = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right)$

$\angle C = \cos^{-1}\left(\frac{\overrightarrow{CA}\cdot\overrightarrow{CB}}{\|\overrightarrow{CA}\|\cdot\|\overrightarrow{CB}\|}\right) = \cos^{-1}\left(\frac{1}{\sqrt{50}}\right)$

④ A fly completes a circle of radius 3cm in 2 seconds.
Find $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ for this path.

Radius = 3

ang. ~~ma~~ veloc. $\omega = \frac{2\pi}{2} = \pi$

$\vec{r}(t) = (3\cos(\pi t), 3\sin(\pi t), 0)$

$\vec{v}(t) = (-3\pi\sin(\pi t), 3\pi\cos(\pi t), 0)$

$\vec{a}(t) = (-3\pi^2\cos(\pi t), -3\pi^2\sin(\pi t), 0)$.

⑤ Solve $z^3 = 1$. Express the answers in both rectangular & polar forms.

if $z = re^{i\theta}$ $(r>0)$ & $z^3 = 1$

then $z^3 = r^3 e^{3i\theta} = 1$

$\Rightarrow$ $r = 1$ & 3θ is a mult. of 2π.

$\Rightarrow$ θ could be $0, \frac{2\pi}{3}$, or $\frac{4\pi}{3}$.

polar	rect.
1	1
$e^{2\pi i/3}$	$-\frac{1}{2} + \frac{\sqrt{3}}{2}i$
$e^{4\pi i/3}$	$-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

(three solutions)

(6) Find a 2^{nd} order linear homog. diffEq which has

$$f(t) = 5e^{-t} - 6e^{-2t}$$

as a solution.

Char. polynomial must have roots $-1, -2$

$\Rightarrow$ it is $(\lambda+1)(\lambda+2)$

$= \lambda^2+3\lambda+2.$

so diffEq. is

$$\boxed{f''(t)+3f'(t)+2f(t)=0}$$

(7) For which values of C do the solutions to

$$f''(t)+8f'(t)+C\cdot f(t)=0$$

char. poly $\lambda^2+8\lambda+C$

involve sines & cosines?

this occurs when char. poly. has nonreal roots.

The roots are $\lambda = \frac{-8\pm\sqrt{8^2-4C}}{2}$; so this occurs when

$8^2-4C<0$, ie. $C > 8^2/4$, ie. $\boxed{C>16}$

(8) Evaluate:

a) $$\sum_{n=1}^{\infty} n\cdot x^n = x\cdot\sum_{n=1}^{\infty} n\cdot x^{n-1} = x\cdot\frac{d}{dx}\left(\sum_{n=1}^{\infty} x^n\right) = x\cdot\frac{d}{dx}\left(\frac{x}{1-x}\right) = \boxed{\frac{x}{(1-x)^2}}$$

b) $$\sum_{n=1}^{\infty}\frac{1}{n}x^n = \int_0^x\left(\sum_{n=1}^{\infty} t^{n-1}\right)dt = \int_0^x\frac{1}{1-t}\,dt$$

$$= \boxed{-\ln(1-x)} \quad \left(\text{or } \ln\left(\frac{1}{1-x}\right)\right)$$