Note: there will not be class on Friday, October 24. You may either submit your homework at your conference session or place it in my mailbox on the first floor of the Kassar building by 3pm.

1. Solve the following initial value problem.

$$\begin{aligned} f'(t) &= 4f(t)^2 t \\ f(0) &= 1 \end{aligned}$$

2. Solve the following initial value problem.

$$\begin{aligned} \frac{dy}{dx} &= 6 \cdot \frac{x^3}{y-1} \\ y(0) &= 3 \end{aligned}$$

3. Solve the following initial value problem.

$$\begin{aligned} \frac{dy}{dt} &= e^{-2y}\sqrt{1-t^2} \\ y(0) &= 1 \end{aligned}$$

4. Solve the following initial value problem.

$$\begin{aligned} f'(x) &= \frac{(1+f(x)^2)}{\sqrt{4-x^2}} \\ f(0) &= 1 \end{aligned}$$

5. Solve the following initial value problem.

$$\begin{aligned} y''(t) + 5y'(t) + 6y(t) &= 6 \\ y(0) &= 1 \\ y'(0) &= 1 \end{aligned}$$

6. Solve the following initial value problem.

$$\begin{aligned} f''(t) + 4f'(t) + 13f(t) &= 104 \\ f(0) &= 11 \\ f'(0) &= 0 \end{aligned}$$

7. Find a path $\vec{r}(t)$ that satisfies the following conditions. Here $\vec{v}(t)$ denotes the velocity and $\vec{a}(t)$ denotes the acceleration.

$$\begin{aligned} \vec{a}(t) &= -\vec{r}(t) \\ \vec{r}(0) &= (1,0,0) \\ \vec{v}(0) &= (0,2,3) \end{aligned}$$

*Hint.* Think of the $x$ coordinate, $y$ coordinate, and $z$ coordinate as three different functions, and solve an initial value problem for each one.

8. Evaluate $\int_0^\infty xe^{-x^2}\,dx$.

9. Evaluate $\int_0^\infty e^{-x}\cos(2x)\,dx$.

   The remaining problems will review summation notation (also called $\Sigma$ notation), which we will need to use extensively in the upcoming unit on series. If this notation is not familiar from your algebra and precalculus courses, there are many useful resources online (search for "summation notation" or "sigma notation"), or ask me or the TAs about it.

10. Evaluate each of the following sums (in this problem, completely simplify your answer).

    (a) $\sum_{n=1}^{5}(2n-1)$

    (b) $\sum_{\ell=0}^{3}(\ell+2)^2$

    (c) $\sum_{k=0}^{5}(-1)^k\cdot 2^{-k}$

    (d) $\sum_{n=-2}^{100} 5$

11. Express each of the following sums in $\Sigma$ notation. You do not need to evaluate the sum.

    (a) $1+2+3+\cdots+99+100$

    (b) $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{20!}$

    (c) $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\cdots+\frac{1}{101}$

    (d) $\frac{15}{2^0}+\frac{14}{2^1}+\frac{13}{2^2}+\cdots+\frac{1}{2^{14}}+\frac{0}{2^{15}}$

12. Compute the sum $\sum_{k=1}^{n}\frac{1}{k(k+1)}$ for $n=1$, $n=2$, $n=3$, and $n=4$ (in this problem, completely simplify your answer). From these computations, make a guess about what the value of this sum is in general (your answer will be in terms of $n$).