Please staple your assignment!

As usual, you may assume that the strength of gravity is either $g = 9.8$ or $g = 10$ meters per second squared, or you may express your answer in terms of g.

1. A frog is sitting on a lily pad at $(10, 40, 0)$ where lengths are measued in centimeters. A fly is currently at $(-20, 40, 10)$, flying in a straight line with velocity vector $(30, 40, 0)$ (measured in centimeters per second).

   (a) Parameterize the path of the fly by giving its position vector $\vec{r}(t)$ as a function of t.

   (b) At what time is the fly closest to the frog?

2. An airplane cruising at 250 meters per second (that is 900 km/hr, the cruising speed of a Boeing 777) is turning to the left. The plane banks 30 degrees to the left to make this turn. How long (in seconds) does it take the airplane to turn 180 degrees (that is, until it is facing the opposite direction), assuming that the pilot is correctly "stepping on the ball?" [1]

   You may answer this question in whatever fashion you wish. The following four steps are a suggestion, but you are not required to do it exactly this way.

   (a) Suppose that the plane completes a semicircle of radius R. Parameterize the path of the airplane, as a function of t. That is, give a function $\vec{r}(t)$ which gives the position vector of the airplane t seconds after it has begun turning. Your answer will be in terms of R, which is a number to be determined later.

   (b) Calculate the g-force during this turn (remember that the g-force vector is given by $\vec{f}(t) = \vec{a}(t) + g\hat{k}$). Your answer will still be in terms of R.

   (c) Compute the angle that the g-force vector makes with $\hat{k}$, again in terms of R.

   (d) Set your answer to the previous part equal to 30 degrees and solve for R. Deduce from this the amount of time it takes to complete the turn.

3. If a projectile is launched with velocity $\vec{v}_0$ at time $t = 0$ from an initial position $\vec{r}_0$, then neglecting air resistance the object's path obeys the following equation (until it hits something else).

$$\vec{r}(t) = \vec{r}_0 + t \cdot \vec{v}_0 - \frac{1}{2}gt^2\hat{k}$$

   (a) Compute the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of such a projectile.

   (b) An archer stands on a hill at $\vec{r}_0 = (0, 0, 10)$ and fires an arrow with initial velocity $\vec{v}_0 = (30, 0, 40)$. Assume that the ground at the foot of the hill is at $z = 0$. Find the coordinates of the point where the arrow will hit the ground.

[1] The phrase "step on the ball" means that the pilot banks the plane correctly so that the g-force is perpendicular to the floor. The phrase refers to the fact that a passenger should be able to stand on a ball in the cabin without the ball rolling.

4. A particle is moving along a path parameterized by $\vec{r}(t) = (12t^2, 5\cos(\pi t^2), 5\sin(\pi t^2))$.
   (a) Describe or sketch the path the particle is following.
   (b) Compute the velocity, speed, and acceleration of the particle, as functions of t.
   (c) Compute the total length of the path followed by the participle between $t = 0$ and $t = 2$.

5. A particle is moving along a path parameterized by $\vec{r}(t) = (t + \cos t, \sin t, t)$.
   (a) Describe or sketch the path the particle is following.
   (b) Compute the velocity, speed, and acceleration of the particle, as functions of t.
   (c) Write an integral giving the length of the path followed by the particle between $t = 0$ and $t = 10$ seconds. Use Wolfram Alpha (or any other computing system) to compute this length (do not attempt to compute it by hand).

6. Convert the following points from polar to rectangular coordinates.
   (a) $(r, \theta) = (7, \pi/3)$
   (b) $(r, \theta) = (11, 0)$
   (c) $(r, \theta) = (28, \pi/6)$
   (d) $(r, \theta) = (2, \pi/2)$

7. Convert the following points from rectangular to polar coordinates.
   (a) $(7, 7)$
   (b) $(5, 12)$
   (c) $(0, -24)$
   (d) $(-7\sqrt{3}, -7)$

8. Express the equation $x^2 - 2x + y^2 - 2y = 0$ in the form $r = f(\vartheta)$, for some function $f(\vartheta)$.

9. Express the equation $r = 4\sin\vartheta \cdot \cos\vartheta$ in rectangular coordinates.

10. Given a function $f(x)$, the following numbers, which measure how much $f(x)$ resonates with various waves, are called *Fourier coefficients.*

$$
\begin{aligned}
A_n &= \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\, dx \quad (\text{for } n = 0, 1, 2, 3, \ldots) \\
B_n &= \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)\, dx \quad (\text{for } n = 1, 2, 3, \ldots)
\end{aligned}
$$

   (a) Find all Fourier coefficients of the function $f(x) = \sin(7x)$.
   (b) Find the Fourier coefficients A_0, A_1, A_2, B_1, and B_2 of the function $f(x) = x$.