Clearly write your conference section number (C01, C02, or C03) on your problem set, and place it in the correct pile. Also please remember to staple your assignment.

All physical quantities will be expressed in SI units (meters, kilograms, seconds, newtons, joules, Kelvin) if unspecified. You are free to omit the units from your calculations.
For problems involving gravity near the earth, you may leave your answer in terms of g, or you may assume that g is either 9.8 or 10 meters per second squared.

1. Suppose that an ideal spring exerts of force of 25 Newtons when it is stretched 3 meters from equilibrium. Compute the amount of work necessary to stretch the spring an additional 2 meters.

2. Recall from class that the force exerted by the gas in an ideal piston during adiabatic expansion is $F(x) = kx^{-\gamma}$, where k and γ are constants. Suppose that when $x = 1$ meter this force is 5 newtons, and when $x = 0.5$ meter this force is 20 newtons. Compute the work done by the piston when the gas pushes the piston from $x = 2$ to $x = 3$.

3. Suppose that the piston mentioned in the previous problem has cross-sectional area A square meters (which is constant), and length $x(t)$ at time t. Denote the volume of the chamber by $V(t)$, the pressure of the gas by $P(t)$, and the force exerted by the gas on the piston by $F(t) = kx(t)^{-\gamma}$ (you may assume that k, γ have the same values as in the previous problem, or you may leave them as variables). Then the following relations must hold.

$$V(t) = A \cdot x(t)$$
$$P(t) \cdot A = F(t)$$

   The *ideal gas law* states that $P(t)V(t) = nRT(t)$, where $T(t)$ is the temperature of the gas at time t and n, R are constants. Assume as in the previous problem that $F(t) = kx(t)^{-\gamma}$, where k, γ are constants.

   (a) Express the temperature $T(t)$ in terms of the function $x(t)$ and the constants in the problem statement.
   (b) Show that the change in temperature $T(b) - T(a)$ from time $t = a$ to $t = b$ is directly proportional to the amount of work done by the gas on the piston.

4. We mentioned in class that, in the equation $F = kx^{-\gamma}$ for the force of an adiabatic piston, it is always the case that $\gamma > 1$. Suppose instead that $\gamma = 1$.

   (a) Compute the work done by the gas as the piston expands from $x = 1$ to $x = b$.
   (b) What happens in the limit as $b \to \infty$? Explain why this is not physically reasonable.

5. A 100 kg object hangs on the end of a 100 meter cable. The cable itself weighs 0.4 kilograms per meter. Compute the work necessary to wind up the cable (lifting both the object and the cable to the top).

6. Suppose that a triangle has vertices at the three points $(3, 0, 0)$, $(0, 4, 0)$, $(0, 0, 5)$. Compute the lengths of the edges of the triangle, and the measures of the three angles in the triangle.

7. A hiker stands on the summit of one mountain, and observes the summit of another. Let $\vec{v}$ be the vector from the summit where the hiker stands to the other summit (the x axis points east, the y axis ponts north, and the z axis points straight up). The hiker measures that the vector $\vec{v}$ has the following direction cosines.

$$\cos\theta_x = 12/13 \quad \cos\theta_y = 4/13 \quad \cos\theta_z = 3/13$$

The hiker knows that the altitude of the second summit is 3 kilometers higher than the altitude of the current summit. What is the magnitude of the vector $\vec{v}$ (that is, what is the distance between the summits in a straight line)?

8. A control tower is located at the point $(-0.5, 0, 0.2)$ (lengths are in kilometers), observing an airplane taking off. The airplane leaves lifts off at the point $(0, 2, 0)$ at time $t = 0$ and then travels to the point $(0, 14, 5)$ at a constant speed of 0.2 kilometers per second. What is the distance from the control tower to the airplane during this time? Express your answer as a function of t.

9. Suppose that $\vec{F} = (3, 2, 0)$ and $\vec{A} = (1, -1, 2)$. Find two vectors $\vec{F}_1, \vec{F}_2$ such that $\vec{F}_1$ is parallel to $\vec{A}$, $\vec{F}_2$ is perpindicular to $\vec{A}$, and $\vec{F}_1 + \vec{F}_2 = \vec{F}$.

10. Evaluate $\displaystyle\int_{1/2}^{\sqrt{3}/2} \arcsin(x)\ dx$.

11. Find a function $f(x)$ satisfying the following properties.

    - $\displaystyle\int_0^{2\pi} f(x)\sin(2x)dx = 3\pi$
    - $\displaystyle\int_0^{2\pi} f(x)\cos(x)dx = 4\pi$
    - $\displaystyle\int_0^{2\pi} f(x)^2 dx = 25\pi$

    Your answer should be a single function which satisfies all three conditions.