**Terminology:**
- A series $\sum c_n$ is absolutely convergent if $\sum |c_n|$ converges.
- A series $\sum c_n$ is conditionally convergent if $\sum c_n$ converges and $\sum |c_n|$ diverges.

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$$

is an example of a conditionally convergent series.

- Else, $\sum c_n$ is divergent.

**Alternating Series Test**

"Alternating means to go "+", "-", "+", "-"," and so on.

A series $\sum_{n=1}^{\infty} (-1)^n c_n$ (with $c_n > 0$), such that

- the $c_n$ are decreasing (don't overshoot walls)
- $\lim_{n \to \infty} c_n = 0$

then it converges (walls close all the way in)

eg. \[ \text{Recall! } \tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \text{ when the series?} \]

**Question:** When does this converge?

(1) Use ratio test:

$$L = \lim_{n \to \infty} \left| \frac{(-1)^{n+1} \frac{x^{2n+3}}{2n+3}}{(-1)^n \frac{x^{2n+1}}{2n+1}} \right| = L = \lim_{n \to \infty} \left| \frac{x^2 \cdot \frac{2n+1}{2n+3}}{1} \right| = |x^2|$$
Series converges absolutely when \(|x^2| < 1\) (i.e. \(-1 < x < 1\))
otherwise, it diverges

\[ \text{CHECK SPECIFIC VALUES: } x = -1, \ x = 1 \]

When \(x = -1\) converges
\[
\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}
\]

Since \(\frac{1}{2n+1}\) is decreasing
and \(\lim_{n \to \infty} \frac{1}{2n+1} = 0\), this
converges (alternating series test) \(\text{BUT CONVERGENCE}\)
(because absolute value diverges)

Series \(\sum_{n=0}^{\infty} (-1)^n \frac{2n+1}{2n+1}\)

\[ \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \Rightarrow \ln(x+1) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \text{ : When does it converge?} \]

1. Ratio Test: \(L = \lim_{n \to \infty} \left| \frac{(-1)^{n+1} \cdot x^{n+2}}{(-1)^n \cdot x^{n+1}} \right| = \lim_{n \to \infty} \left| \frac{x^{n+2}}{x^{n+1}} \cdot \frac{n+1}{n+2} \right| = 1 \times 1 \)

So when \(|x| < 1\) it converges absolutely, and when \(|x| > 1\) it diverges.

2. When \(x = -1\) diverges
\[ \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} \]

\[= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{n+1} = -1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \ldots \text{, which diverges } (-\infty) \]

Series \(\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}\)