\[ \sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \text{ converges?} \]

Know \[ \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges} \]

 III. 3. 14

Recap:

\[ \sum_{n=a}^{\infty} C_n \rightarrow \text{does this converge?} \]

*Integral test*

\[ \sum_{n=1}^{\infty} \frac{1}{n^p} \]

\[ \text{converge if } p > 1 \]

\[ \text{diverge if } p \leq 1 \]

*aka \( p \)-series*

*Comparison test*

\[ \sum_{n=1}^{\infty} \frac{1}{n^{1.1}} \text{ converges} \rightarrow \text{yes because } 0 < \frac{1}{n^{2.1}} < \frac{1}{n^2} \]

and \[ \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ converges} \]

\[ \sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \]

Compare to \[ \sum_{n=1}^{\infty} \frac{1}{n} \text{ which diverges} \]

but \[ \frac{1}{n} \neq \frac{1}{n+\sqrt{n}} \]

Note that \( \sqrt{n} < n \) (if \( n > 1 \))

\[ n + \sqrt{n} \leq 2n \]

\[ \frac{1}{n+\sqrt{n}} > \frac{1}{2n} \]

So \[ 0 < \frac{1}{2n} < \frac{1}{n+\sqrt{n}} \]

and \[ \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges} \]

So \[ \sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \text{ diverges} \]