INTEGRATION BY PARTS

\[ \frac{d}{dx}(x \cdot e^{2x}) = 1 \cdot e^{2x} + 2x \cdot e^{2x} \rightarrow \text{product rule} \]

\[ \Rightarrow \int (e^{2x} + 2xe^{2x}) \, dx = x \cdot e^{2x} + c \]

\[ \Rightarrow \int xe^{2x} \, dx \]

\[ u = x, \quad dv = e^{2x} \]
\[ du = dx, \quad v = \frac{1}{2} e^{2x} \]

\[ \int xe^{2x} \, dx = \frac{1}{2} xe^{2x} - \frac{1}{2} \int e^{2x} \, dx \]

\[ = \frac{1}{2} xe^{2x} - \frac{1}{4} e^{2x} + C \]

\[ \text{Formula: } \int u(x)v'(x) \, dx = u(x)v(x) - \int u'(x)v(x) \, dx \]

\[ = \int u \, dv = uv - \int v \, du \]

\[ \Rightarrow \int x \cos x \, dx \]

\[ u = x, \quad dv = \cos x \]
\[ du = dx, \quad v = \sin x \]

\[ \int x \cos x \, dx = x \sin x - \int \sin x \, dx \]

\[ x \sin x + C \]

\[ \int e^{3x} \sin(3x) \, dx \]

\[ u = 3 \cos 3x, \quad dv = e^{3x} \]
\[ du = -9 \sin 3x \, dx, \quad v = \frac{1}{3} e^{3x} \]

\[ \int e^{3x} \sin(3x) \, dx = \frac{1}{9} \left( 2e^{3x} \sin 3x - 36 \int e^{3x} \sin(3x) \, dx \right) \]

\[ \Rightarrow \int e^{3x} \sin 3x \, dx = \frac{1}{37} \left( 2e^{3x} \sin 3x - 12e^{3x} \cos 3x \right) \]
Order of preference for what is "u"

\[ L \rightarrow \text{logarithms} \]
\[ I \rightarrow \text{inverse trig functions} \]
\[ A \rightarrow \text{algebraic functions (} x, x^n, \frac{1}{x}, \sqrt{x} \text{)} \]
\[ T \rightarrow \text{trig functions} \]
\[ E \rightarrow \text{exponentials} \]

\[ \text{Inverse trig.} \]
\[ \int \tan^{-1} x \, dx \]
\[ u = \tan^{-1} x \quad dv = dx \]
\[ du = \frac{1}{1+x^2} \quad v = x \]

\[ \left[ x \tan^{-1} x \right]_0^1 - \int_0^1 x \cdot \frac{1}{1+x^2} \, dx \]
\[ \left[ x \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x}{1+x^2} \, dx \]
\[ \frac{\pi}{4} - \frac{1}{2} \left[ \ln |1| \right]^2 \]
\[ \frac{\pi}{4} - \frac{1}{2} \ln 2 \]

\[ u = 1 + x^2 \quad du = 2x \, dx \]
\[ x \, dx = \frac{1}{2} \, du \]

new boundaries
\[ 1 + x^2 \rightarrow 1, 2 \]

\[ \int \sin^m x \cos^n x \, dx \]

Two tactics \( u \) substitution

- \( u = \sin x \) or \( u = \cos x \)
- \( \sin^2 x + \cos^2 x = 1 \)
- \( \sin^2 x = \frac{1}{2} (1 - \cos 2x) \)
- \( \cos^2 x = \frac{1}{2} (1 + \cos 2x) \)

\[ \int \sin^2 x \, dx \rightarrow \int \frac{1}{2} (1 - \cos 2x) \, dx \]