MATH 19
FINAL EXAM
14 DECEMBER 2014

Name : ____

Show all of your reasoning. You may use the back of each page for additional space or scratch work. You do not need to simplify your answers unless specifically instructed to do so.

You may use one page of notes (front and back). You do not need to submit it with the exam. No other calculators or aids are permitted.

1	/25	**2**	/10	**3**	/10
4	/10	**5**	/10	**6**	/10
7	/10	**8**	/10	**9**	/10
10	/15	**11**	/15	**12**	/15
				Σ	/150

(1) **Short answer questions.** You do not need to show any work for the following questions.

(a) Evaluate $\int_0^\infty e^{-x/3}\,dx$. Answer: __________

(b) Determine whether each series converges or diverges.

$\sum_{n=1}^{\infty} 1$	$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	$\sum_{n=1}^{\infty} \frac{1}{n}$	$\sum_{n=1}^{\infty} \frac{1}{n^2}$
☐ converges	☐ converges	☐ converges	☐ converges
☐ diverges	☐ diverges	☐ diverges	☐ diverges
$\sum_{n=1}^{\infty}(-1)^{n-1}$	$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$	$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$	$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$
☐ converges	☐ converges	☐ converges	☐ converges
☐ diverges	☐ diverges	☐ diverges	☐ diverges

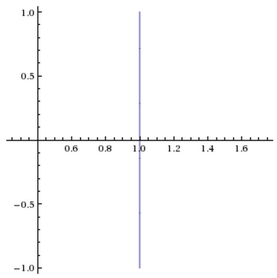
(c) Find the angle between the following two vectors. You do not need to simplify your answer.

$$\begin{aligned} \vec{v} &= (1,1,1) \\ \vec{w} &= (1,-1,1) \end{aligned}$$

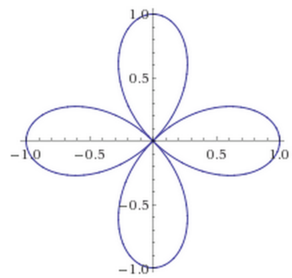
Answer: __________

(d) Evaluate $\sum_{n=0}^{\infty} \frac{2^n}{n!}$. Answer: __________

(e) Identify which polar equation describes each graph (circle the appropriate letter).



A B C D



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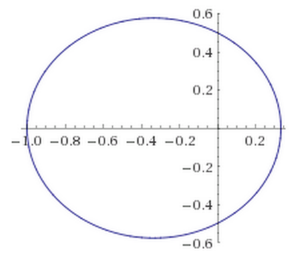

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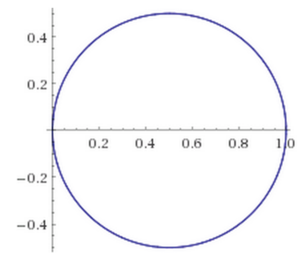

A B C D

$$A: \quad r = \cos\theta$$
$$B: \quad r = \cos(2\theta)$$
$$C: \quad r = \frac{1}{\cos\theta}$$
$$D: \quad r = \frac{1}{2+\cos\theta}$$

(f) Determine the three complex Fourier coefficients c_{-1}, c_0, and c_1 of the following function.

$$f(x) = 2\cos x - 6\sin x$$

$c_{-1} =$ _____ $c_0 =$ _____ $c_1 =$_____

(g) Determine the radius of convergence of the following series.

$$\sum_{n=0}^{\infty} 2^n x^n$$

Answer: __________

(2) Solve the following initial value problem.

$$\begin{aligned} \frac{dy}{dx} &= y \cdot \cos(2x) \\ y(0) &= 2 \end{aligned}$$

(3) (a) Find the quadratic approximation of the function $f(x) = (1+x)^{100}$ around the center $x = 0$.

(b) Use this to approximate the value of 1.001^{100}.

(4) Let $\vec{f}$ and $\vec{v}$ be the following vectors.

$$\begin{aligned} \vec{f} &= (7, 8, 19) \\ \vec{v} &= (1, 0, 1) \end{aligned}$$

Find two vectors $\vec{f_1}, \vec{f_2}$ such that $\vec{f_1}$ is parallel to $\vec{v}$, $\vec{f_2}$ is perpendicular to $\vec{v}$, and $\vec{f} = \vec{f_1} + \vec{f_2}$.

(5) Evaluate the following sum where it converges (as a function of x).

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k \cdot x^{2k}}$$

(6) Find complex numbers z, w satisfying the following two equations.

$$\begin{aligned}(1+2i)w + (3-4i)z &= 30 \\ 2w + z &= 5\end{aligned}$$

(7) Find a series of rational numbers whose sum converges to the value of the following integral.

$$\int_0^1 x \cdot e^{-x^3}\, dx$$

(8) Let $f(t)$ be a function defined by the following Fourier series.

$$f(t) = \sum_{n=1}^{\infty} \frac{1}{2^n} \sin(nt)$$

(a) Show that this series converges for all values of t by using appropriate facts and convergence tests.

(b) Determine the values of the following integrals. You do not need to show any work for full credit.

$$\int_{-\pi}^{\pi} f(t)\ dt \quad = \quad \underline{\hspace{4cm}}$$

$$\int_{-\pi}^{\pi} f(t)\sin(3t)\ dt \quad = \quad \underline{\hspace{4cm}}$$

$$\int_{-\pi}^{\pi} f(t)\cos(7t)\ dt \quad = \quad \underline{\hspace{4cm}}$$

(9) Evaluate $\int_0^2 x^2\sqrt{4 - x^2}\ dx$.

(10) An airplane takes off from the point $(2, 0, 0)$ at time $t = 0$ and travels to the point $(-2, 8, 4)$ at constant speed over the course of 40 seconds. An observer stands at the point $(0, 2, 0)$ watching the plane's flight.

(a) Determine the position $\vec{r}(t)$ of the airplane at time t.

(b) What is the location of the airplane at the moment that it is closest to the observer?

(11) Find a vector function $\vec{r}(t)$ satisfying the following conditions (here $\vec{v}(t)$ and $\vec{a}(t)$ denote the velocity and acceleration, respectively).

$$\begin{aligned}\vec{a}(t) &= -2\vec{v}(t) - 2\vec{r}(t)\\ \vec{r}(0) &= (0, 1, -1)\\ \vec{v}(0) &= (1, 0, 1)\end{aligned}$$

(12) Let $f(t)$ be the steady-state solution (the unique solution that is 2π-periodic) to the following differential equation.

$$f''(t) + 2f'(t) + f(t) = 2\sin(2t) - 4\cos(3t)$$

(a) Determine the complex Fourier coefficients of the function $V(t) = 2\sin(2t) - 4\cos(3t)$.

(b) Determine the complex Fourier coefficients of $f(t)$.

(continues on next page)

(c) Write $f(t)$ as a real Fourier series (in terms of sines and cosines). Simplify your answer enough that there are no imaginary numbers in the expression.

Additional space for work