Note: Due to the midterm on Friday 10/5, this assignment is slightly shorter than usual. I recommend spending some time working reviewing old problems, working the “suggested” problems from earlier sets, and making sure you understand the theorems and proofs from class.

Read: The rest of §8.

Suggestion: Work (or think about) the following problems. Problems marked with a * have answers given at the back of the book.

- §8 : 3*, 5, 10

1. (a) Let \( f \in S_n \) be the cycle \((x_1, x_2, \cdots, x_r)\). Show that \( o(f) = r \).

(b) Suppose that \( f = (x_1, x_2, \cdots, x_r) \circ (y_1, y_2, \cdots, y_s) \). Assume that these are disjoint cycles (that is, \( x_i \neq y_j \) for all \( i, j \)). Prove that the order of \( f \) is the least common multiple of \( r \) and \( s \).

(c) Find two transpositions whose product has order 3. This shows that the “disjoint” hypothesis is essential in part (b).

For the next two exercises: Read the statement of Saracino exercise 8.10(a). You may use this statement without proof (but it is a good review exercise to prove it yourself).

2. Determine the largest possible order of an element of \( S_9 \).

3. Does \( A_6 \) have an element of order 6? Does \( A_7 \)? If so, give an example. If not, prove that it is impossible.

4. Suppose that \( H \) is a subgroup of \( S_n \). Prove that either all elements of \( H \) are even permutations, or exactly half of the elements of \( H \) are even permutations.

   Hint: Mimic the proof from class on Friday 9/28 that exactly half of the elements of \( S_n \) are in \( A_n \).

5. Read the description of the dihedral group \( D_n \) of order \( 2n \) in Saracino Exercise 8.15. Solve parts (a) and (b) of that problem (check your answer to (b) in the back of the book).