0. Look over the list of potential project topics posted on the course website (under handouts), and think about which ones you might be most interested in, as well as who you might like to work with in your group. I'll be sending out a link to a poll early next week to state your preferences, to be submitted by Wednesday evening. I may continue to add project topics to the list as I think of more or hear suggestions.

1. Describe an action of the circle group S^1 on 3-dimensional space $\mathbb{R}^3$ (an informal description is fine, as long as it is clear what you mean). Describe the orbits and stabilizers of your action, and comment on their dimensions.

2. (**10.1.2**) Let $D_8 = \langle a, b \mid a^4 = b^2 = e, ba = a^3 b\rangle$, and S_3 be the symmetric group of degree 3. Let $G = D_8 \times S_3$. Let $H = \langle b\rangle \times \langle(1\ 2\ 3)\rangle$ and $K = \langle a\rangle \times \langle(1\ 2\ 3)\rangle$ be subgroups of G. Is H a normal subgroup of G? What about K?

3. (**10.2.7**) **Stabilizers of elements in the same orbit.** Let the group G act on the set Ω. Let $g \in G$, and $\alpha, \beta \in \Omega$. Assume that $g \cdot \alpha = \beta$. Show that $\mathrm{Stab}_G(\beta) = g\mathrm{Stab}_G(\alpha)g^{-1}$. Conclude that two elements in the same orbit have isomorphic stabilizers.

4. (**10.3.1**) Let $G = S_4$ and let $K = \{1, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$. K is a normal subgroup of G. What is the order of G/K? Which group is G/K isomorphic to?

5. (**10.3.2**) Let $G = (\mathbb{Z}/18\mathbb{Z}, +)$ be a cyclic group of order 18.
   (a) Find a subgroup H of G with $|H| = 3$.
   (b) What are the elements of G/H?
   (c) Find a familiar group that is isomorphic to G/H.

6. (**10.3.8**) Assume G is cyclic, and $N \lhd G$. Is G/N cyclic?

7. Suppose that G is a group, H is a normal subgroup of G with $|H| = 7$ and $[G : H] = 20$. Lagrange's theorem implies that if $x \in H$, then $x^7 = e$. Prove the converse: if $x \in G$ satisfies $x^7 = e$, then $x \in H$.

   Hint: use the quotient group.

8. (**11.1.10**) Let $\phi : G \longrightarrow H$ be an onto homomorphism.
   (a) Assume that G is abelian. Does this imply that H is abelian? What about the converse?
   (b) What if we replaced abelian by cyclic in the above question.

9. (**11.3.3**)

   (a) Verify that the mapping $f\colon (\mathbb{R}, +) \longrightarrow (\mathbb{C}^\times, \cdot)$ given by $f(x) = \cos(2\pi x) + i\sin(2\pi x)$ is a homomorphism.

   (b) We know that $\mathbb{R}/\ker(f) \cong \mathrm{Im}(f)$. Explicitly find $\ker(f)$ and $\mathrm{Im}(f)$.

10. (**11.3.4**) By finding an appropriate homomorphism $\phi : \mathrm{GL}(n, \mathbb{R}) \to ?$, show that $\mathrm{SL}(n, \mathbb{R}) \lhd \mathrm{GL}(n, \mathbb{R})$, and find a familiar group that is isomorphic to $\mathrm{GL}(n, \mathbb{R})/\mathrm{SL}(n, \mathbb{R})$.

Some other good problems to try for additional practice (but not to hand in):
10.1.1, 10.1.9, 10.2.4, 10.3.3, 10.3.9, 11.3.5