- **Note:** Due to the midterm on Friday 10/5, this assignment is slightly shorter than usual. I recommend spending some time working reviewing old problems, working the "suggested" problems from earlier sets, and making sure you understand the theorems and proofs from class.
- **Read:** The rest of §8.
- **Suggestion:** Work (or think about) the following problems. Problems marked with a $*$ have answers given at the back of the book.
  - §8 : 3*, 5, 10

1. (a) Let $f \in S_n$ be the cycle $(x_1, x_2, \cdots, x_r)$. Show that $o(f) = r$.

   (b) Suppose that $f = (x_1, x_2, \cdots, x_r) \circ (y_1, y_2, \cdots, y_s)$. Assume that these are *disjoint* cycles (that is, $x_i \neq y_j$ for all i, j). Prove that the order of f is the least common multiple of r and s.

   (c) Find two transpositions whose product has order 3. This shows that the "disjoint" hypothesis is essential in part (b).

- **For the next two exercises:** Read the statement of Saracino exercise 8.10(a). You may use this statement without proof (but it is a good review exercise to prove it yourself).

2. Determine the largest possible order of an element of S_9.
3. Does A_6 have an element of order 6? Does A_7? If so, give an example. If not, prove that it is impossible.
4. Suppose that H is a subgroup of S_n. Prove that either all elements of H are even permutations, or exactly half of the elements of H are even permutations.

   Hint: Mimic the proof from class on Friday 9/28 that exactly half of the elements of S_n are in A_n.
5. Read the description of the *dihedral group* D_n *of order* $2n$ in Saracino Exercise 8.15. Solve parts (a) and (b) of that problem (check your answer to (b) in the back of the book).