Study guide

- (§3.3) Know the definition of dimension. Make sure you understand the definition, and why it captures the intuitive idea of “degrees of freedom.”
- (§3.3) Be familiar with the “standard bases” for $\mathbb{R}^n, \mathcal{P}_d,$ and $M_{2 \times 2}$.
- (§3.4) Know the definition of “coordinates of $\vec{v}$ in basis $B$”, and the shorthand notation $[\vec{v}]_B$.
- (§3.4) If $S$ is the standard basis for $\mathbb{R}^n$, then for all $\vec{v} \in \mathbb{R}^n$, $[\vec{v}]_S = \vec{v}$ (the coordinate vector is the same as the vector itself). Make sure you understand why!
- (§3.4) If you are given a vector $\vec{v}$ and a basis $B$, how do you compute the coordinates $[\vec{v}]_B$?
- (§3.4) Know the definition of the change of basis matrix (also called transition matrices) $[T]_B^B$ and how to compute them. Know the basic facts about inverses and products of change of basis matrices.

Textbook problems

- §3.3: 40 (Hint: write the general solution to $A\vec{x} = \vec{0}$, and express the result as a linear combination.)
- §3.4: 4, 14, 18, 22, 24

Terminology note: the textbook says “ordered basis” where we’ve usually just said “basis.” Also, the phrase “transition matrix” means the same as “change of basis matrix.”

Supplemental problems:

1. Suppose that $B = \{\vec{u}, \vec{v}\}$ is a basis for a vector space $V$. Prove that $\{3\vec{u} + 2\vec{v}, \vec{u} + \vec{v}\}$ is also a basis for $V$.

2. Suppose that $A$ is an invertible $n \times n$ matrix. Prove that the columns of $A$ form a basis for $\mathbb{R}^n$.

3. Suppose that $B$ is an orthonormal basis for a $\mathbb{R}^n$ (see PSet 6 problem 3 for the definition of an orthonormal set. An orthonormal basis is an orthonormal set that is also a basis). Prove that for every $\vec{u} \in \mathbb{R}^n$,

$$\|\vec{u}\| = \|[\vec{u}]_B\|.$$ 

In other words: length is measured the same way in any orthonormal system of coordinates.