Study guide

- (§3.3) Know the definition of dimension. Make sure you understand the definition, and why it captures the intuitive idea of “degrees of freedom.”
- (§3.3) Be familiar with the “standard bases” for \( \mathbb{R}^n \), \( \mathcal{P}_d \), and \( M_{2 \times 2} \).
- (§3.4) Know the definition of “coordinates of \( \vec{v} \) in basis \( B \)”, and the shorthand notation \([\vec{v}]_B\).
- (§3.4) If \( S \) is the standard basis for \( \mathbb{R}^n \), then for all \( \vec{v} \in \mathbb{R}^n \), \([\vec{v}]_S = \vec{v}\) (the coordinate vector is the same as the vector itself). Make sure you understand why!
- (§3.4) If you are given a vector \( \vec{v} \) and a basis \( B \), how do you compute the coordinates \([\vec{v}]_B\)?
- (§3.4) Know the definition of the change of basis matrix (also called transition matrices) \([T]_B^{B'}\) and how to compute them. Know the basic facts about inverses and products of change of basis matrices.

Textbook problems

- §3.3: 40 (Hint: write the general solution to \( A\vec{x} = \vec{0} \), and express the result as a linear combination.)
- §3.4: 4, 14, 18, 22, 24

Terminology note: the textbook says “ordered basis” where we’ve usually just said “basis.” Also, the phrase “transition matrix” means the same as “change of basis matrix.”

Supplemental problems:

1. Suppose that \( B = \{\vec{u}, \vec{v}\} \) is a basis for a vector space \( V \). Prove that \( \{3\vec{u} + 2\vec{v}, \vec{u} + \vec{v}\} \) is also a basis for \( V \).

2. Suppose that \( A \) is an invertible \( n \times n \) matrix. Prove that the columns of \( A \) form a basis for \( \mathbb{R}^n \).

3. Suppose that \( B \) is an orthonormal basis for a \( \mathbb{R}^n \) (see PSet 6 problem 3 for the definition of an orthonormal set. An orthonormal basis is an orthonormal set that is also a basis). Prove that for every \( \vec{u} \in \mathbb{R}^n \),

\[
\|\vec{u}\| = \|\vec{u}_B\|.
\]

In other words: length is measured the same way in any orthonormal system of coordinates.