Study guide

- (§1.8) Understand how to convert a flow/traffic/circuit problem to a linear system of equations.
- (§1.3) Know how to add, multiply, and transpose matrices. Be aware of any restrictions on the dimensions of the matrices involved. Try odd-numbered problems from 1 to 23 in §1.3 to review these (check your answers in the back).
- (§1.3) The formula for matrix multiplication seems very strange at first. Make sure you understand why it is defined the way it is. It helps to think about some examples.
- (Discussed in-class Friday 2/9) A $2 \times 2$ matrix encodes a (linear) transformation of the plane. Given such a matrix $A$, understand how to transform individual points or simple picture (e.g. the unit square). Similarly, understand how to find the matrix $A$ given a picture of its effect.

Textbook problems from DeFranza and Gagliardi:

- §1.8: 8, 20. **Comments about these problems:**
  - There is a misprint in 1.8.20: the 16 V battery (bottom wire) should be facing the other way.
  - In problem 1.8.8, when the authors ask what the smallest possible value of $x_8$ is, they mean *in order for all flow rates to be nonnegative*. The issue is that the linear system of equations doesn’t tell the whole story, because some of its solutions would require negative traffic along some of the (one-way) roads.
  - You may use Mathematica (or other software) to perform any row-reduction in these problems.

- §1.3: 10, 12, 16, 22, 26, 28, 37, 38
  - Hint for 1.3.28: write a linear system of equations in variables $a, b, c, d$ to describe the situation.

Supplemental problems:

1. Find a $2 \times 2$ matrix $A$ such that $A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $A \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$.

2. A $2 \times 2$ matrix $A$ transforms the unit square in the plane in the manner shown below. Determine the matrix $A$ (there is more than one possible answer; you only need to give one).

3. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Draw a pair of pictures like in the problem above to illustrate the way that the matrix $A$ transforms the unit square.