This document includes all of PSet 9, including content covered on Friday 4/12 or later and not covered on the midterm exam. The assignment is longer than usual since it covers two weeks of material.

Study guide

- (lab; see also §6.3)) Understand the *Gram-Schmidt procedure* for turning a basis $B = \{\vec{v}_1, \cdots, \vec{v}_n\}$ into an *orthonormal* basis $\{\vec{u}_1, \cdots, \vec{u}_n\}$.
- (§6.2) Know the definition of an *inner product*, and some examples. Also know the terminology *inner product space.*
- (§6.2) How do you define $\|\vec{v}\|$ and $\vec{u} \perp \vec{v}$ in an inner product space (not necessarily $\mathbb{R}^n$)?
- (§6.2) Understand the proof of the "Pythagorean theorem for inner product spaces."
- (§6.3) Know the definition of the projection $\text{proj}_{\vec{v}}\vec{u}$ in an inner product space, as well as the interpretation as the "nearest multiple" of $\vec{v}$ to $\vec{u}$.

  Midterm material ends here; the rest concerns material covered on Friday 4/12 or later)
- (§4.1) Know the definition of a "linear transformation," and their basic properties.
- (§4.1) Be comfortable with the notation $T: V \to W$.
- (§4.1) Understand why linear transformations $\mathbb{R}^n \to \mathbb{R}^m$ is always representend by an $m \times n$ matrix (and what "represented by" means in this context).
- (§4.2) Given a matrix A, know the definitions of $N(A)$ and $R(A)$, and how to compute bases for both of them (both are reformulations of algorithms studied earlier in the course).
- (§4.2) Know the "rank-nullity theorem." (Theorem 5 in the textbook)

Textbook problems

- §6.2: 2, 4, 10, 12, 29(a,b,c)
- §6.3: 10, 18

  Midterm material ends here.
- §4.1: 26, 30, 39, 42
- §4.2: 22, 32, 40

Supplemental problems:

1. Let V be an inner product space. Prove that if $S = \{\vec{v}_1, \cdots, \vec{v}_n\}$ is a list of vectors, and $\vec{u}$ is another vector such that $\vec{u} \perp \vec{v}_i$ for all i, then $\vec{u}$ is also orthogonal to any linear combination of the elements of S.

2. Suppose that $\{\vec{u}, \vec{v}, \vec{w}\}$ is an orthogonal set in an inner product space V. Prove that for any three scalars $a, b, c \in \mathbb{R}$,

$$\|a\vec{u} + b\vec{v} + c\vec{w}\|^2 = a^2\|\vec{u}\|^2 + b^2\|\vec{v}\|^2 + c^2\|\vec{w}\|^2.$$

   Hint: apply the Pythagorean theorem for inner product spaces twice.