Study guide

- (§3.3) Know the definition of *dimension.* Make sure you understand the definition, and why it captures the intuitive idea of "degrees of freedom."
- (§3.3) Be familiar with the "standard bases" for $\mathbb{R}^n, \mathcal{P}_d$, and $M_{2\times 2}$.
- (§3.4) Know the definition of "coordinates of $\vec{v}$ in basis B", and the shorthand notation $[\vec{v}]_B$.
- (§3.4) If S is the standard basis for $\mathbb{R}^n$, then for all $\vec{v} \in \mathbb{R}^n$, $[\vec{v}]_S = \vec{v}$ (the coordinate vector is the same as the vector itself). Make sure you understand why!
- (§3.4) If you are given a vector $\vec{v}$ and a basis B, how do you compute the coordinates $[\vec{v}]_B$?
- (§3.4) Know the definition of the *change of basis matrix* (also called *transition matrices*) $[I]_B^{B'}$ and how to compute them. Know the basic facts about inverses and products of change of basis matrices.

Textbook problems

- §3.3: 40 (*Hint:* write the general solution to $A\vec{x} = \vec{0}$, and express the result as a linear combination.)

- §3.4: 4, 14, 18, 22, 24

  Terminology note: the textbook says "ordered basis" where we've usually just said "basis." Also, the phrase "transition matrix" means the same as "change of basis matrix."

Supplemental problems:

1. Suppose that $B = \{\vec{u}, \vec{v}\}$ is a basis for a vector space V. Prove that $\{3\vec{u} + 2\vec{v}, \vec{u} + \vec{v}\}$ is also a basis for V.

2. Suppose that A is an *invertible* $n \times n$ matrix. Prove that the columns of A form a basis for $\mathbb{R}^n$.

3. Suppose that B is an *orthonormal basis* for a $\mathbb{R}^n$ (see PSet 6 problem 3 for the defintion of an orthonormal set. An orthonormal basis is an orthonormal set that is also a basis). Prove that for every $\vec{u} \in \mathbb{R}^n$,
$$\|\vec{u}\| = \|[\vec{u}]_B\|.$$
   In other words: length is measured the same way in any orthonormal system of coordinates.