Study guide

- (§3.3) Know the definition of a basis. Understand why both parts (spanning, and linear independence) are included.
- (§3.3) Be able to determine whether a given subset of a specific vector space is a basis.
- (§3.3) Be able to find the basis of a subspace defined by several free variables (e.g. problem 3.3.20).
- (§3.3) Be able to find a basis for a span of a set of vectors in $\mathbb{R}^n$ (e.g. problem 3.3.26).
- (§3.3) Know the definition of *dimension.* Make sure you understand the definition, and why it captures the intuitive idea of "degrees of freedom."
- (§3.3) Be familiar with the "standard bases" for $\mathbb{R}^n, \mathcal{P}_d$, and $M_{2\times 2}$.
- (§3.4) Know the definition of "coordinates of $\vec{v}$ in basis B", and the shorthand notation $[\vec{v}]_B$.
- (§3.4) If S is the standard basis for $\mathbb{R}^n$, then for all $\vec{v} \in \mathbb{R}^n$, $[\vec{v}]_S = \vec{v}$ (the coordinate vector is the same as the vector itself). Make sure you understand why!
- (§3.4) If you are given a vector $\vec{v}$ and a basis B, how do you compute the coordinates $[\vec{v}]_B$?
- (§3.4) Know the definition of the *change of basis matrix* (also called *transition matrices*) $[I]_B^{B'}$ and how to compute them. Know the basic facts about inverses and products of change of basis matrices

Textbook problems

- §3.3: 12, 20, 26, 38, 40
- §3.4: 4, 14, 18, 22, 24

  Terminology note: the textbook says "ordered basis" where we've usually just said "basis." Also, the phrase "transition matrix" means the same as "change of basis matrix."

Supplemental problems:

1. Suppose that A is an $n \times n$ matrix. Let $W \subseteq M_{n\times n}$ denote the set of matrices B such that $AB = BA$ (that is, A and B commute).
   (a) Show that W is a *subspace* of $M_{2\times 2}$.
   (b) Suppose that A is not a scalar multiple of the identity matrix. Prove that W is at least 2-dimensional (Hint: show that it contains the span of $\{A, I\}$).
2. Suppose that $B = \{\vec{u}, \vec{v}\}$ is a basis for a vector space V. Prove that $\{3\vec{u} + 2\vec{v}, \vec{u} + \vec{v}\}$ is also a basis for V