Note: This assignment covers two weeks of material, to avoid having an assignment due in an exam week. Therefore it is longer than usual.

Textbook problems from DeFranza and Gagliardi:

> *Suggestion:* Do the odd-numbered problems from 1 to 15 in §4.1, and check your answer in the back of the book. Try to develop a good intuition for what is linear and what is not.

- §4.1: 22, 26, 30, 39, 42, 44
- §4.2: 6, 16, 22, 32, 38, 40

Midterm 2 material ends here

- §4.3: 8, 18, 28, 30
- §4.4: 14, 20, 30, 36

Supplemental problems:

Any supplemental problems will not concern material on the exam, and will be posted after it.

1. (Polynomial interpolation). This sequence of problems concerns the following situation: given a list of data points $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$, is it possible to fit a polynomial equation $y = p(x)$ of degree d to these points?

   (a) Suppose that n *distinct* numbers $x_1, \cdots, x_n$ are chosen, and consider the linear transformation
   $$\mathcal{P}_{n-1} \to \mathbb{R}^n$$
   given by $T(p(x)) = \begin{pmatrix} p(x_1) \\ p(x_2) \\ \vdots \\ p(x_n) \end{pmatrix}$. Prove that T is an isomorphism.

   Hint: you may use, without proof, the following fact from algebra: if $p(x)$ is a nonzero polynomial of degree d, then $p(x) = 0$ can have at most d solutions.

   (b) Deduce from (a) that given a list of n data points $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$, where all of the x-coordinates are distinct, there is always a *unique* polynomial $p(x)$ of degree $n - 1$ passing through all of them (i.e. $p(x_i) = y_i$ for each i).

   (c) Find the unique cubic (degree 3) polynomial $p(x)$ such that $y = p(x)$ passes through $(-1, -11), (0, -7), (1, -3)$, and $(2, 7)$.

2. Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by reflection across the line $y = 7x$.

   (a) Find a basis B in which the matrix representation of T is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

   (b) Use your answer from (a) to determine the matrix representation of T in the standard basis.

(Same "important notes" and "submission instructions" apply as before; omitted to save space.)