

**Textbook problems** from DeFranza and Gagliardi:

- §1.4: 2, 6, 8, 11, 22
- §1.5: 14, 18, 26, 30

**Supplemental problems:**

1. Suppose that the matrix equation  $A\vec{x} = \vec{b}$  has a unique solution. Prove that the homogeneous equation  $A\vec{x} = \vec{0}$  has no nontrivial solutions.
2. Suppose that  $A$  is an invertible matrix. Prove that the transpose  $A^t$  is also invertible, and that its inverse is given by  $(A^{-1})^t$ .

*Hint:* You may wish to make use of some algebraic facts about transposition, summarized in Theorem 6 on page 36.

3. (a) Let  $A$  be an  $n \times n$  matrix. Prove that if the numbers in each row of  $A$  sum to 0, then  $A$  is not invertible.

*Hint:* Use Corollary 5 from Friday's handout.

- (b) Deduce from part (a) and problem 2 that if instead *the numbers in each column* of a matrix  $A$  all sum to 0, then  $A$  is not invertible.

*Note:* To clarify the wording: part (a) concerns matrices like  $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 1 & -4 & 3 \end{pmatrix}$  are not invertible

(note that in each of the three rows, the numbers sum to 0), while part (b) concerns matrices

like  $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & -4 \\ -3 & -1 & 3 \end{pmatrix}$ .

**Important notes:**

- Regrade requests must be submitted via Gradescope within *one week of the due date* of the assignment.
- For full credit, you must show or explain your reasoning.
- You are encouraged to work in groups while solving the problems, but all submitted work must be your own work in your own words. Use of solution manuals or online solution databases is plagiarism, and will result in a 0 on the assignment in addition to being reported to Community Standards.

**Submission instructions:**

Before submitting your assignment scan it to a single pdf file and **view your pdf to make sure that it is clearly legible**. Then submit it as follows.

1. Go to <http://www.gradescope.com> and log in.
2. Select "Math 272" and the appropriate homework assignment, then select "submit pdf."
3. For each written question, select the pages of your submission where your solution appears.
4. Click submit.