1. [9 points] Solve the following system of linear equations.

\[
\begin{align*}
    x_2 + x_3 + x_4 &= 5 \\
    x_1 + 3x_3 + 7x_4 &= 14 \\
    x_1 + 2x_3 + 5x_4 &= 11
\end{align*}
\]

2. [9 points] Recall that two matrices \(A, B\) commute if \(AB = BA\). Consider the following three matrices.

\[
A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}
\]

(a) Determine whether \(A\) and \(B\) commute. (b) Determine whether \(B\) and \(C\) commute. (c) Determine whether \(A\) and \(C\) commute.

3. [9 points] (a) Suppose that \(A\) is an \(n \times n\) matrix. Show that if \(A\) is invertible, then \(A\vec{x} = \vec{0}\) has no nontrivial solutions \(\vec{x}\).

(b) Using part (a), show that \(A = \begin{pmatrix} 1 & 1 & -2 \\ 2 & -3 & 1 \\ -1 & -1 & 2 \end{pmatrix}\) is not an invertible matrix. (\textit{Hint:} the numbers in each row sum to 0.)

4. [9 points] The augmented matrix of a linear system has the form

\[
\begin{pmatrix}
    1 & 2 & -1 & a \\
    2 & 3 & -2 & b \\
    -1 & -1 & 1 & c
\end{pmatrix}
\]

(a) Determine the values of \(a, b, c\) for which this linear system is consistent.

(b) For those values of \(a, b, c\) for which the system is consistent, does it have a unique solution or infinitely many solutions? Briefly explain why.

5. [9 points] Write a system of linear equations that describes the traffic flow pattern for the network in the figure. \textbf{You do not need to solve the system.}

6. [9 points] Consider the following three vectors.

\[
\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}
\]

(a) Show that \(\vec{v}_1, \vec{v}_2, \vec{v}_3\) are linearly independent.
(b) Find the unique scalars \( c_1, c_2, c_3 \) such that the vector

\[
\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}
\]

is equal to \( c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 \).