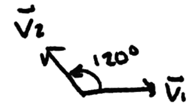
example of linear transformation: isometric drawings.

Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}$.

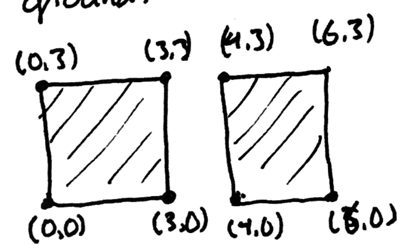


One can define a linear transformation $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ as follows:

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x \cdot \vec{v}_1 + y \cdot \vec{v}_2 + z \cdot \left(\tfrac{1}{2}\vec{v}_1 + \vec{v}_2\right).$$

This transformation can be used to draw 3D objects on the page.

e.g. suppose two buildings have these footprints on the ground:



The one on the left is 8 units tall; the one on the right is 10 units tall. So, e.g., the eight corners of the first building are:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 8 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix}.$$

Applying T, we obtain locations to draw these in the plane. We can then work out which faces are "visible" where, and make a drawing (at left).