

Matrices & linear systems (cf. §1.2)

9/8/17

linear system $\left\{ \begin{array}{l} X_2 - X_3 + 3X_4 + 4X_5 = 33 \\ 2X_1 + X_2 + 7X_3 + 3X_4 - X_5 = 22 \\ -2X_1 - 8X_3 - X_5 = -31 \\ 4X_1 - X_2 + 17X_3 - 3X_4 + 7X_5 = 92 \end{array} \right.$

4 equations in 5 variables.

$$\begin{bmatrix} 0 & 1 & -1 & 3 & 4 \\ 2 & 1 & 7 & 3 & -1 \\ -2 & 0 & -8 & 0 & -1 \\ 4 & -1 & 17 & -3 & 7 \end{bmatrix}$$

coefficient matrix
4x5

$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 3 & 4 & 33 \\ 2 & 1 & 7 & 3 & -1 & 22 \\ -2 & 0 & -8 & 0 & -1 & -31 \\ 4 & -1 & 17 & -3 & 7 & 92 \end{array} \right]$$

augmented matrix
4x6
(vertical line optional)

an example of
row echelon form

$$\begin{bmatrix} \textcircled{2} & 1 & 7 & 3 & -1 & 22 \\ 0 & \textcircled{1} & -1 & 3 & 4 & 33 \\ 0 & 0 & 0 & 0 & \textcircled{-6} & -42 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

("pivots" are circled)

"pivot" of a row: first nonzero entry.

- All "zero rows" at bottom.
- Pivots go from left to right.

an example of
reduced row echelon form

$$\begin{bmatrix} \textcircled{1} & 0 & 4 & 0 & 0 & 12 \\ 0 & \textcircled{1} & -1 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- all pivots are 1.
- all other entries in a pivot's column are 0.

The three elementary row operations :

1) Swap two rows.

eg. $\begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 4 \\ 3 & 0 & 7 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 0 \\ 3 & 0 & 7 \end{pmatrix}$

2) Multiply a row by a nonzero constant.

eg. $\begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 0 \\ 3 & 0 & 7 \end{pmatrix} \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 3 & 0 & 7 \end{pmatrix}$

3) Add a multiple of one row to another.

eg. $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 3 & 0 & 7 \end{pmatrix} \xrightarrow{R_3 - \frac{1}{3}R_1 \rightarrow R_3} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Theorem Performing elem. row operations on an augmented matrix does not change the set of solutions of the linear system corresponding to the augmented matrix.