Supplement to problem set 8
(Concerns material from §4.3)

1. Suppose that $T: V \to V$ is a linear operator. Prove that T is injective if and only if T is surjective.

2. Suppose that $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$ are vectors in a vector space V, and $T: V \to W$ is a linear transformation to another vector space W.

   (a) Suppose that $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$ are linearly independent and T is injective. Prove that $T(\vec{v}_1), T(\vec{v}_2), \cdots T(\vec{v}_n)$ are linearly independent.

   (b) Suppose that $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$ span V, and that T is surjective. Prove that $T(\vec{v}_1), T(\vec{v}_2), \cdots T(\vec{v}_n)$ span W.

   (c) Deduce from (a) and (b) that if $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$ is a basis for V, and T is an isomorphism, then $T(\vec{v}_1), T(\vec{v}_2), \cdots T(\vec{v}_n)$ is a basis for W.