### Suggested reading for this week (from the textbook): §1.1, §1.2, §1.3

### Study items for PSet 1:
- Proof by contradiction that $\sqrt{2} \notin \mathbb{Q}$.
- Related irrationality proofs, e.g. $\sqrt{2} + 1 \notin \mathbb{Q}$.
- Sets and related notation, including $\in$, $\subseteq$, and $\emptyset$.
- Set-builder notation.
- How to work with sets of sets, e.g. indexed collections.
- Set operations: $\cap$, $\cup$, $\setminus$, and $\times$.
- Visualizing set operations using Venn diagrams.
- Unions and intersections of indexed collections; the notation $\bigcup_{i \in I} S_i$ and $\bigcap_{i \in I} S_i$.
- Statement and proof of de Morgan’s laws for sets.
- Be aware that set complement notation is only defined when a “universe” $U$ has been chosen.
- Definition and properties of the “power set” $\mathcal{P}(S)$.

### Problems from the book:
- Section 1.1: 1, 3, 9
  (in 9(b), it is sufficient to answer “true” or “false;” no explanation is needed)
- Section 1.2: 1

### Supplemental problems:
1. Prove that $2\sqrt{2}$ is irrational. You may assume the fact that $\sqrt{2}$ is irrational (proved in class).

2. The following paragraph claims to prove that $\sqrt{4}$ is irrational, following the same strategy that we used in class for $\sqrt{2}$ and $\sqrt{3}$. This cannot be correct, since $\sqrt{4} = 2$. Identify the specific step in the argument where the error occurs, and explain why that step is not valid.

   **Suppose, for contradiction, that $\sqrt{4}$ is rational.** Then by expressing it as a reduced fraction, we may write $\sqrt{4} = \frac{a}{b}$ for some integers $a$ and $b$ with no common factors. It follows that $4b^2 = a^2$. The left side of this equation, $4b^2$, is divisible by 4. Therefore the right side, $a^2$, is also divisible by 4. It follows that $a$ is divisible by 4. Thus $a = 4c$ for some integer $c$, and so $4b^2 = (4c)^2 = 16c^2$. Cancelling a factor of 4, we obtain $b^2 = 4c^2$. Therefore $b^2$ is divisible by 4, and by the same logic as above, $b$ itself is divisible by 4. But this shows that both $a$ and $b$ are divisible by 4, which is a contradiction. Therefore $\sqrt{4}$ is not rational.

3. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. For each set expressed in set-builder notation below, list all the elements of the set (write the elements within curly braces).
Problem Set 1

(a) $S = \{x \in U \mid x^2 < 46\}$
(b) $S = \{x \in U \mid x^2 - 3x = -2\}$

4. Let $V = \{-3, -2, -1, 0, 1, 2, 3\}$. Which of the following sets are equal?

(a) $A = \{n \in V \mid |n| < 2\}$
(b) $B = \{n \in V \mid n^2 \leq 1\}$
(c) $C = \{n \in V \mid n^3 = n\}$
(d) $D = \{-1, 0, 1\}$
(e) $E = \{n \in V \mid n^2 \leq n\}$

5. Define two sets as follows.

\[
\begin{align*}
A & = \{1\} \\
B & = \left\{ \{1\}, \\{\{1\}\}, \\{\{1\}, \{\{1\}\}\} \right\}
\end{align*}
\]

For each of the following statements, determine where it is true or false. Briefly explain your answer.

(a) $A \in B$
(b) $A \subseteq B$
(c) $\{A\} \in B$
(d) $\{A\} \subseteq B$