NP

Goal of this worksheet: use logical symbols to extract the underlying structure from some statements in English.

Work on the problems below with one or two students nearby. **This does not need to be handed in, and you don't need to finish all the problems during class.** Call me over if you have questions or want to check answers!

Don't worry if some of this seems unfamiliar, or if some questions seem a bit ambiguous. The worksheet is an exercise to help you learn the material and think about **new** things. It is not a test, and you don't need to be able to do all of it right away.

1. Let A and B denote the following two statements (in terms of variables a and b).

$$\begin{aligned} A &= \text{“}a \text{ is even”} \\ B &= \text{“}b \text{ is even”} \end{aligned}$$

Write each of the following statements using A, B, and the logical connectives $\sim, \wedge, \vee$.

(a) "a is odd."

$\sim A$

(b) "Both a and b are even."

$A \wedge B$

(c) "At least one of a and b is odd."

$(\sim A) \vee (\sim B)$

(d) "a and b have the same parity." (The word "parity" refers to a number's status as either even or odd.)

$(A \wedge B) \vee \left[(\sim A) \wedge (\sim B)\right]$ or $(A \wedge B) \vee (\sim A \wedge \sim B)$

(with usual order of operations: $\sim, \wedge, \vee$)

(e) "Exactly one of a and b is even."

(even $A \wedge B \vee \sim A \wedge \sim B$ is technically ok but it could be confusing!)

$(A \wedge \sim B) \vee (\sim A \wedge B)$

2. Write each of the following statements using logical quantifiers ($\exists$ and/or $\forall$) and logical connectives (you may not need logical connectives in all of them). I haven't specified the basic statements here like in the previous problem, so you'll need to fill some in.

all of these have other ways to write them as well.

   (a) "An integer is never both odd and even."

   $\forall n \in \mathbb{Z},\ \sim\big((n \text{ is odd}) \vee (n \text{ is even})\big)$ <u>OR</u> $\sim\exists n \in \mathbb{Z}$ st. $\big[(n \text{ is odd}) \vee (n \text{ is even})\big]$

   (b) "The equation $x = \cos x$ has a real solution."

   $\exists x \in \mathbb{R}$ st. $x = \cos x$

   (c) "No real number has a negative square."

   $\forall x \in \mathbb{R},\ \sim(x^2 < 0)$ <u>OR</u> $\sim\exists x \in \mathbb{R}$ st. $x^2 < 0$.

   (d) "There are no even prime numbers other than 2."

   $\sim\exists p \in \mathbb{N}$ st. $\big[\sim(p = 2) \wedge (p \text{ is prime})\big]$

3. (Challenge) Express the following statement using logical quantifiers: "there is no largest integer."

   $\sim\exists n \in \mathbb{Z}$ st. $\big(\forall m \in \mathbb{Z},\ m \leq n\big)$

   <u>OR</u> $\forall n \in \mathbb{Z},\ \exists m \in \mathbb{Z}$ st. $m > n$.

4. (Challenge) Express the following statement using logical quantifiers: "there is a *unique* real solution to the equation $x = \cos x$."

   $\exists x \in \mathbb{R}$ st. $\Big[(x = \cos x) \wedge \big(\cancel{\nexists} \sim\exists y \in \mathbb{R} \text{ st. } (\sim(x = y) \wedge \cos y = y)\big)\Big]$