Goal of this worksheet: use logical symbols to extract the underlying structure from some statements in English.

Work on the problems below with one or two students nearby. **This does not need to be handed in, and you don't need to finish all the problems during class.** Call me over if you have questions or want to check answers!

Don't worry if some of this seems unfamiliar, or if some questions seem a bit ambiguous. The worksheet is an exercise to help you learn the material and think about **new** things. It is not a test, and you don't need to be able to do all of it right away.

1. Let A and B denote the following two statements (in terms of variables a and b).

$$\begin{aligned} A &= \text{“}a \text{ is even”} \\ B &= \text{“}b \text{ is even”} \end{aligned}$$

Write each of the following statements using A, B, and the logical connectives $\sim, \wedge, \vee$.

(a) “a is odd.”

(b) “Both a and b are even.”

(c) “At least one of a and b is odd.”

(d) “a and b have the same parity.” (The word “parity” refers to a number's status as either even or odd.)

(e) “Exactly one of a and b is even.”

2. Write each of the following statements using logical quantifiers ($\exists$ and/or $\forall$) and logical connectives (you may not need logical connectives in all of them). I haven't specified the basic statements here like in the previous problem, so you'll need to fill some in.

   (a) "An integer is never both odd and even."

   (b) "The equation $x = \cos x$ has a real solution."

   (c) "No real number has a negative square."

   (d) "There are no even prime numbers other than 2."

3. (Challenge) Express the following statement using logical quantifiers: "there is no largest integer."

4. (Challenge) Express the following statement using logical quantifiers: "there is a *unique* real solution to the equation $x = \cos x$."