

Suggested reading for this week (from the textbook): §3.2 (Euclidean algorithm), §3.3 (Prime factorization)

Study items for PSet 5:

- Statement of the pigeonhole principle.
- Understanding pigeonhole arguments as “worst-case” arguments.
- The “sums of contiguous subsets” trick (from the worksheet and examples in the text).
- Be comfortable explaining the pigeonhole arguments from the worksheet and from class.
- Definition of $a \mid b$.
- Basic divisibility facts and their proofs (e.g. Lemmas 1 and 2 from Friday 2/28).
- The division algorithm and its proof.
- General structure of existence and uniqueness proofs.
- Using the Euclidean algorithm to compute $\gcd(a, b)$ and express it as $au + bv$.
- Statement of the Fundamental Theorem of Arithmetic, and proof of the “existence” part.
- Statement of Euclid’s lemma.

Problems from the book: (First two numbers refer to the section number)

- 2.3.1 (12 eggs being dyed 5 colors)
- 2.3.2 (Number of cards needing to be drawn; four parts)
- 2.3.10 (50 whole numbers and subsets summing to multiples of 25)
- 2.3.11 (Must nine whole numbers have a subset summing to a multiple of 10?)
- 3.1.2 (Some divisibility implications; two parts)
- 3.1.3 (Does $a \mid bc$ always imply $a \mid b$ or $a \mid c$?)
- 3.1.6, parts (a) and (b) (Set-theoretic formulations of $a \mid b$)
- 3.1.10 (Remainders of primes when divided by 6)
- 3.1.17 (Remainders when divided by 7; three parts)
- 3.1.23 ($k^n - 1$ is divisible by $k - 1$)
- 3.2.1 (Euclidean algorithm examples; six parts)

Supplemental problems:

1. (a) Prove that if $x \in \mathbb{R}$ satisfies $x > 1$, then for all $n \in \mathbb{N}$ there exists an integer $m \in \mathbb{N}$ such that $x^m > n$. (One slick way to do this is to make use of Bernoulli’s inequality, proved on last week’s problem set). Try to make your proof as elementary as possible, e.g. avoiding using facts about logarithms or topics from calculus.
(b) Use part (a) to deduce that if $x \in \mathbb{R}$ satisfies $0 < x < 1$, then for all $n \in \mathbb{N}$ there exists an integer $m \in \mathbb{N}$ such that $x^m < \frac{1}{n}$. (You may assume that the claim in (a) is true whether or not you have solved part (a) already.)