Note: I'm also planning to release instructions for the second entry in the Proof Portfolio in the next couple days. I'll update this document to include these instructions once they are prepared. This entry will not be due until Friday May 1 (the last day of class), and the rest of the portfolio will not be due until finals week.

UPDATE: the proof portfolio information is now posted in a separate document. Submitting the second entry by May 1 is now optional. There is also a LaTeX template posted that you may use for the portfolio if you wish.

Suggested reading for this week (from the textbook): §8.1 (sequences)

Problems from the book: (First two numbers refer to the section number)

- 6.2.5 (images and inverse images for the ceiling function; four parts)
- 6.2.9 ($C \subseteq D \Rightarrow f(C) \subseteq f(D)$; does the converse hold?)
- 6.2.10 ($f(C \cup D) = f(C) \cup f(D)$)
- 6.2.12 ($C \subseteq D \Rightarrow f^{-1}(C) \subseteq f^{-1}(D)$; does the converse hold?)
- 6.2.13 ($f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$)
- 6.3.1 (Having the same cardinality is reflexive, symmetric, and transitive)
- 6.3.2 ($\mathbb{Z}$ is countable. This was done in class but it is good practice to carefully write out the argument.)
- 6.3.3 (comparing cardinality of some intervals; three parts)
- 6.3.6 (the set of functions from $\{1, 2\}$ to $\mathbb{N}$ is countable)
- 6.3.9 (how to consolidate three "Hilbert Hotels")

Supplemental problems:

1. Suppose that A, B, C, D are four sets (possibly infinite!) such that $|A| = |B|$ and $|C| = |D|$. Prove that $|A \times C| = |B \times D|$.
2. (a) Define $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ by the formula $f(m, n) = 2^m 3^n$. Prove that f is injective, but not surjective.
   (b) Find an injective function $\mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}$. (Your function need not be surjective, but it is fine if it is).
3. (Bonus, for a small amount of extra credit) Find a bijective function $f : [0, 1] \to (0, 1]$.

   Comment: I said in the videos this week that I would put something on the homework about an explicit formula for the bijection $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ described in the video. I decided it was probably a little too technical to use as a homework problem, but if you are interested, here is a formula for that function:

   $$f(m, n) = \binom{m+n-1}{2} + m.$$

   It is an interesting (and tricky!) excercise to prove that this function is bijective; you may enjoy giving it a try. But I've decided not to assign it officially.