Proof Portfolio, Entry 1

Your name here

Remarks in blue are comments about the assignment; you should not reproduce them in your write-up.

Try to recreate all text in black exactly as-is, including all formatting. Feel free to reword the prose if you wish, however. The purpose is to practice basic formatting and symbols in LaTeX. Then fill in the proofs. You should also feel free to add any expository remarks, transition sentences, etc. that you feel will make the document easier to follow.

A general comment: you'll see in some of the text below sentences like "We will now prove..." and the like. This follows a common convention in written mathematics to use the first person plural pronoun "we." The theory is that math is always cooperative, with the author and reader working through all notions together. This convention may strike you as bizarre, and you should feel free to change "we" to "I" or choose a different style entirely, according to your preference.

Introduction

The purpose of this document is to establish the irrationality of certian class of real numbers, namely roots of prime numbers. First recall some basic facts and definitions.

1. A real number a is called *rational* if there exist integers m, n with $n \neq 0$ and $a = \frac{m}{n}$.

2. (*Bézout's identity*) If a and b are two integers, not both 0, then there exist integers u, v such that
$$au + bv = \gcd(a, b).$$
Furthermore, if there exist $u, v \in \mathbb{Z}$ such that $au + bv = 1$, then $\gcd(a, b) = 1$.

3. (*Euclid's lemma*, weak form) If p is a prime number and a, b are integers such that $p \mid ab$, then either $p \mid a$ or $p \mid b$.

1 Preliminary results

We now prove two preliminary lemmas.

Lemma 1.1. *If a is a rational number, then there exist integers m, n such that $a = \frac{m}{n}$ and $\gcd(m, n) = 1$.*

Proof. Fill this in yourself. You may follow your notes from class verbatim if you wish, but feel free to reorganize or reword according to your own style and taste. The same remark applies in the two proofs below. □

Lemma 1.2. *If p is a prime number, and a, n are natural numbers such that $p \mid a^n$, then $p \mid a$.*

Proof. Fill this in. □

2 Irrationality of kth roots of primes

With the preliminaries in place, we move on to the main theorem.

Theorem 2.1. *If p is a prime number and k is an integer with $k \geq 2$, then $\sqrt[k]{p} \notin \mathbb{Q}$.*

Proof. Fill this in. □