Study items for PSet 1:

- Proof by contradiction that $\sqrt{2} \not\in \mathbb{Q}$.
- Related irrationality proofs, e.g. $\sqrt{2} + 1 \not\in \mathbb{Q}$.
- Sets and related notation, including $\in$, $\subseteq$ and $\emptyset$.
- Set-builder notation.
- How to work with sets of sets, e.g. indexed collections.
- Set operations: $\cap$, $\cup$, $\setminus$, and $\times$.
- Visualizing set operations using Venn diagrams.
- Unions and intersections of indexed collections; the notation $\bigcup_{i\in I} S_i$ and $\bigcap_{i\in I} S_i$.
- Statement and proof of de Morgan’s laws for sets.
- Be aware that set complement notation is only defined when a “universe” $U$ has been chosen.
- Definition and properties of the “power set” $\mathcal{P}(S)$.

Study items for PSet 2:

- Definition of a “partition.”
- Logical symbols: $\lor$, $\land$, $\sim$, $\Rightarrow$, $\Leftrightarrow$.
- Standard translations, e.g. “$P \Rightarrow Q$” = “$\sim P \lor Q$” and “$P \Leftrightarrow Q$” = “$(P \Rightarrow Q) \land (Q \Rightarrow P)$”.
- Quantifiers $\forall$, $\exists$ and their meaning.
- Be able to fluently translate mathematical statements from English to logic notation and vice versa.
- Negating students; syntax for “moving a negation through” quantifiers and logical symbols. Make sure you understand why these rules hold!
- Terms: “converse” and “contrapositive.” The contrapositive is equivalent to the original statement, while the converse is not.
- Direct proofs of universal statements (“Let $x$ be any element of $S$...”)
- Direct proofs of existential statements (“Let $x = [something specific]...’”)
- Direct proofs of implications (“Assume $P$... therefore $Q$”)
- Definitions of “even” and “odd” in terms of existence of an integer $k$.

Study items for PSet 3:

- Truth tables, and their use in proving tautologies or that logical expressions are equivalent.
- Proving biconditionals in two parts (“$\Rightarrow$” ... “$\Leftarrow$”)
- Direct proofs
Problem Set Study Items

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- Indirect proofs: the distinction between proofs by contradiction and proof by contrapositive.

Study items for PSet 4:

- Know the statement of the “principle of mathematical induction.”
- Proof by induction (ordinary form)
- Proof by strong induction
- Proving formulas or inequalities for sums via induction.
- Vocabulary: “base case,” “inductive hypothesis,” “inductive step.”
- How to recognize when a problem is well-suited to an inductive approach.

Study items for PSet 5:

- Statement of the pigeonhole principle.
- Understanding pigeonhole arguments as “worst-case” arguments.
- The “sums of contiguous subsets” trick (from the worksheet and examples in the text).
- Be comfortable explaining the pigeonhole arguments from the worksheet and from class.
- Definition of $a \mid b$.
- Basic divisibility facts and their proofs (e.g. Lemmas 1 and 2 from Friday 2/28).
- The division algorithm and its proof.
- General structure of existence and uniqueness proofs.
- Using the Euclidean algorithm to compute $\gcd(a, b)$ and express it as $au + bv$.
- Statement of the Fundamental Theorem of Arithmetic, and proof of the “existence” part.
- Statement of Euclid’s lemma.

Study items for PSet 6:

- Know how to prove: $\gcd(a, b) = 1$ if and only if $\exists u, v \in \mathbb{Z}$ such that $au + bv = 1$.
- Know the two formulations of “Euclid’s lemma” (for relatively prime integers, and for prime integers), and why one implies the other.
- The proof of the “uniqueness” half of the fundamental theorem of arithmetic.
- Some ways to use the fundamental theorem of arithmetic: divisibility in terms of prime factorization; greatest common divisors; least common multiples.
- Definition of binomial coefficients $\binom{n}{k}$.

Study items for PSet 7:

- Statement and proof of the binomial theorem.
• Statement and examples of the fundamental counting principle.

• Various examples of dividing counting problems into “tasks” to count with FCP.

• Permutations and factorials, and counting problems where they arise.

• The formula \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \) and its proof via FCP.

• Understand when to add and when to multiply in counting problems.

• Definitions: experiment, sample space, outcome, event, probability.

**Study items for PSet 8:**

• Computing probabilities of various poker hands.

• Set-theoretic definition of “function.”

• Definitions: domain, target set, image/range.

• Definitions: injective (one-to-one), surjective (onto), bijective.

• Proof organization for a typical proof of injectivity and/or surjectivity.

• Definition of function composition.

**Study items for PSet 9:**

• What it means for two functions to be equal.

• Relationship between function composition and injective/surjective/bijective.

• Inverse functions in terms of graphs (and examples).

• Inverse functions, definition in terms of function composition.

• Definition: the identity functions id\(_S\) on a set \(S\).

• Know how to prove: a function has an inverse iff it is bijective.

• Uniqueness of inverse functions.

**Study items for PSet 10:**

• Definitions: image and inverse image of a set.

• How image and inverse image relate to set containment, intersection, and union.

• Examples of a set that is in bijection with a subset of itself.

• Hilbert Hotel (various versions: two or three hotels, or infinitely many).

• Definition of \(|A| = |B|\) (“same cardinality”) for arbitrary sets (not necessarily finite).

• Bijections \(\mathbb{Z} \rightarrow \mathbb{N}\) and \(\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}\).

• Definition: countable and uncountable. Examples of both.
• Proof that $\mathbb{R}$ is uncountable.

• Bijections $\mathbb{R} \to (0, \infty)$ and $\mathbb{R} \to (a, b)$.

Study items for content not on PSet 10 but possibly on midterm:

• Definition of $|A| \leq |B|$ for arbitrary sets (not necessarily finite). Examples.

• Statement of the Cantor-Schröder-Bernstein Theorem (but not the proof!), and some applications.