Study items for PSet 1:

- Proof by contradiction that $\sqrt{2} \notin \mathbb{Q}$.
- Related irrationality proofs, e.g. $\sqrt{2} + 1 \notin \mathbb{Q}$.
- Sets and related notation, including $\in, \subseteq$ and $\emptyset$.
- Set-builder notation.
- How to work with sets of sets, e.g. indexed collections.
- Set operations: $\cap, \cup, \setminus, ^c$, and $\times$.
- Visualizing set operations using Venn diagrams.
- Unions and intersections of indexed collections; the notation $\bigcup_{i\in I} S_i$ and $\bigcap_{i\in I} S_i$.
- Statement and proof of de Morgan's laws for sets.
- Be aware that set complement notation is only defined when a "universe" U has been chosen.
- Definition and properties of the "power set" $\mathcal{P}(S)$.

Study items for PSet 2:

- Definition of a "partition."
- Logical symbols: $\vee, \wedge, \sim, \Rightarrow, \Leftrightarrow$.
- Standard translations, e.g. "$P \Rightarrow Q'' =$ "$\sim P \vee Q''$ and "$P \Leftrightarrow Q'' =$ "$(P \Rightarrow Q) \wedge (Q \Rightarrow P)''$.
- Quantifiers $\forall, \exists$ and their meaning.
- Be able to *fluently* translate mathematical statements from English to logic notation and vice versa.
- Negating students; syntax for "moving a negation through" quantifiers and logical symbols. Make sure you understand *why* these rules hold!
- Terms: "converse" and "contrapositive." The contrapositive is equivalent to the original statement, while the converse is not.
- Direct proofs of universal statements ("Let x be any element of S...")
- Direct proofs of existential statements ("Let $x =$ [something specific]...')
- Direct proofs of implications ("Assume P... therefore Q")
- Definitions of "even" and "odd" in terms of existence of an integer k.

Study items for PSet 3:

- Truth tables, and their use in proving tautologies or that logical expressions are equivalent.
- Proving biconditionals in two parts ("$\Rightarrow''$ . . . "$\Leftarrow''$)
- Direct proofs

- Indirect proofs: the distinction between proofs by contradiction and proof by contrapositive.

Study items for PSet 4:

- Know the statement of the "principle of mathematical induction."
- Proof by induction (ordinary form)
- Proof by strong induction
- Proving formulas or inequalities for sums via induction.
- Vocabulary: "base case," "inductive hypothesis," "inductive step."
- How to recognize when a problem is well-suited to an inductive approach.

Study items for PSet 5:

- Statement of the pigeonhole principle.
- Understanding pigeonhole arguments as "worst-case" arguments.
- The "sums of contiguous subsets" trick (from the worksheet and examples in the text).
- Be comfortable explaining the pigeonhole arguments from the worksheet and from class.
- Definition of $a \mid b$.
- Basic divisibility facts and their proofs (e.g. Lemmas 1 and 2 from Friday 2/28).
- The division algorithm and its proof.
- General structure of existence and uniqueness proofs.
- Using the Euclidean algorithm to compute $\gcd(a, b)$ and express it as $au + bv$.
- Statement of the Fundamental Theorem of Arithmetic, and proof of the "existence" part.
- Statement of Euclid's lemma.

Study items for PSet 6:

- Know how to prove: $\gcd(a, b) = 1$ if and only if $\exists u, v \in \mathbb{Z}$ such that $au + bv = 1$.
- Know the two formulations of "Euclid's lemma" (for relatively prime integers, and for prime integers), and why one implies the other.
- The proof of the "uniqueness" half of the fundamental theorem of arithmetic.
- Some ways to use the fundamental theorem of arithmetic: divisibility in terms of prime factorization; greatest common divisors; least common multiples.
- Definition of *binomial coefficients* $\binom{n}{k}$.

Study items for PSet 7:

- Statement and proof of the binomial theorem.

- Statement and examples of the fundamental counting principle.
- Various examples of dividing counting problems into "tasks" to count with FCP.
- Permutations and factorials, and counting problems where they arise.
- The formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and its proof via FCP.
- Understand when to add and when to multiply in counting problems.
- Definitions: experiment, sample space, outcome, event, probability.

Study items for PSet 8:

- Computing probabilities of various poker hands.
- Set-theoretic definition of "function."
- Definitions: domain, target set, image/range.
- Definitions: injective (one-to-one), surjective (onto), bijective.
- Proof organization for a typical proof of injectivity and/or surjectivity.
- Definition of function composition.

Study items for PSet 9:

- What it means for two functions to be equal.
- Relationship between function composition and injective/surjective/bijective.
- Inverse functions in terms of graphs (and examples).
- Inverse functions, definition in terms of function composition.
- Definition: the identity functions id_S on a set S.
- Know how to prove: a function has an inverse iff it is bijective.
- Uniqueness of inverse functions.

Study items for PSet 10:

- Definitions: image and inverse image of a set.
- How image and inverse image relate to set containment, intersection, and union.
- Examples of a set that is in bijetion with a subset of itself.
- Hilbert Hotel (various versions: two or three hotels, or infinitely many).
- Definition of $|A| = |B|$ ("same cardinality") for arbitrary sets (not necessarily finite).
- Bijections $\mathbb{Z} \to \mathbb{N}$ and $\mathbb{N} \times \mathbb{N} \to \mathbb{N}$.
- Definition: countable and uncountable. Examples of both.

- Proof that $\mathbb{R}$ is uncountable.
- Bijections $\mathbb{R} \to (0, \infty)$ and $\mathbb{R} \to (a, b)$.

Study items for content not on PSet 10 but possibly on midterm:

- Definition of $|A| \leq |B|$ for arbitrary sets (not necessarily finite). Examples.
- Statement of the Cantor-Schröder-Bernstein Theorem (but not the proof!), and some applications.