Amherst College
Department of Mathematics and Statistics

MATH 220 MIDTERM 2 SPRING 2020

NAME:____

Read This First!

- The exam is due on Gradescope at **Wednesday, April 29 at 10pm Eastern time**.
- The exam is open-book and open-notes. You may also freely consult my online lecture notes and anything on the course webpage.
- You may not discuss the problems with anyone except the official course staff (instructor, Math Fellow, and Q Center staff). The course staff can clarify questions and can look over your work, but will not provide detailed hints.
- You may not ask these questions on any websites or search for solutions online.
- Please read each question carefully. Show **ALL** work clearly.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- You may cite any theorems proved in class or on the homework in your proofs, except in cases where the statement to be proved is essentially the same as a theorem proved earlier. In that case you should write out the full proof. Please ask me if you are uncertain about whether you should prove a theorem or if it is enough to cite it.
- **Please write solutions to each problem on a separate page. Include all of your scratchwork in your scanned document.** Any pages of scratchwork that you do not want graded should still be included; put these at the back of your scanned file, and label them as "scratchwork."

Grading - For Instructor Use Only

Question:	1	2	3	4	5	Total
Points:	12	12	12	12	12	60
Score:						

0. Write out the following sentence, and sign your name: "The attached work is entirely my own. I have not discussed the problems with anyone except course staff, and have not attempted to find solutions online." (This page does not need to be labelled with a problem on Gradescope.)

1. Let a and b be two positive integers. This problem concerns a slight variation on Bézout's identity.
   (a) [6 points] Prove that if $\gcd(a, b) \mid 12$, then there exist $u, v \in \mathbb{Z}$ such that $au + bv = 12$.
   (b) [6 points] Prove conversely that if there exist $u, v \in \mathbb{Z}$ such that $au + bv = 12$, then $\gcd(a, b) \mid 12$.

2. Let A, B, C be three sets, and suppose that $f : A \to B$ and $g : B \to C$ are two functions.
   (a) [5 points] Prove that if $g \circ f$ is bijective, then g is surjective.
   (b) [5 points] Prove that if $g \circ f$ is bijective, then f is injective.
   (c) [2 points] Give an example showing that it is possible that $g \circ f$ is bijective, but neither f nor g is bijective. It is enough to state the example (no proof is required).

3. Let A and B be two sets.
   (a) [3 points] Give a definition of the *identity function* id_A. Be sure to state its domain and target set.
   (b) [9 points] Suppose that $f : A \to B$ and $g : B \to A$ are two functions such that $g \circ f = \mathrm{id}_A$. Let h be the function $f \circ g$. Prove that $h \circ h = h$.

4. [12 points] Let $f : A \to B$ be a function, and let S, T be two subsets of B such that $S \subseteq T \subseteq B$. Prove that
$$f^{-1}(T \setminus S) = f^{-1}(T) \setminus f^{-1}(S).$$

5. [12 points] The town of Binomialopolis has 100 residents, of whom 10 plan to vote for a certain ballot proposition in the upcoming election. The other 90 plan to vote against the proposition. A pollster wishes to measure public support for the proposition. To do so, she chooses five (different) residents at random, and asks them whether or not they plan to vote for the proposition.

   (a) Determine the probability that none of the five polled residents plan to vote for the proposition. You do not need to simplify your answer.
   (b) Determine the probability that a majority of the five polled residents plan to vote for the proposition. You do not need to simplify you answer.

   Comment: Calculations like these are important in determining how much confidence can be placed in the accuracy of a poll result, when only a small number of people from a large population are polled. Real polls use a much larger sample than five people, of course!

Please remember to include all your scratchwork in your scanned document, even pages you do not wish to have graded. These pages do not need to be labelled to a problem when submitting on Gradescope.