Math 121 **Midterm Exam #1** **February 21, 2014**

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $\sinh(\ln 3)$, $e^{\ln 4}$, $\ln(e^7)$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points]

(a) Let $y = \arcsin x$. Use implicit differentiation to **PROVE** that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$.

(b) From part (a) we now know that $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$. You may use this fact to **PROVE** that

$$\int \frac{1}{\sqrt{9-x^2}}\,dx = \arcsin\left(\frac{x}{3}\right) + C \qquad \longleftarrow \textbf{Prove this.}$$

2. [30 Points] Evaluate each of the following **limits**. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to 0} \frac{5xe^x - \arctan(5x)}{\sinh x + \ln(1-x)}$

(b) $\displaystyle\lim_{x\to\infty} \left(e^{\frac{1}{x}} - \frac{4}{x}\right)^x$

(c) $\displaystyle\lim_{x\to\infty} (\ln x)^{\frac{3}{x}}$

3. [30 Points] Compute the following **definite integral**. Please simplify your answer.

(a) $\displaystyle\int_0^{\ln 7} x\sinh x \; dx$

(b) $\displaystyle\int_3^{3\sqrt{3}} \frac{1}{\sqrt{36-x^2}} + \frac{1}{9+x^2}\;dx$

(c) $\displaystyle\int_1^e \frac{1}{x[1+(\ln x)^2]}\;dx$

4. [30 Points] Compute the following **indefinite integral**.

(a) $\displaystyle\int x \arcsin x \; dx$

(b) $\displaystyle\int \frac{e^x}{(e^{2x}+4)^{\frac{7}{2}}} \; dx$

(c) $\displaystyle\int \ln(x^2+1) \; dx$

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the following **indefinite integral**.

1. $\displaystyle\int e^{\sqrt{1+\sqrt{x}}} \; dx$

OPTIONAL BONUS #2 Compute the following **indefinite integral**.

2. $\displaystyle\int \frac{\ln(x-1)}{\sqrt{x}} \; dx$