

You may skip crossed out problems, which concern material that won't be

Review Packet for Exam #2 on our exam.

(Section 1, Spring 2018)

Math 121-D. Benedetto

Integrals: Compute each of the following integrals, or else show that it diverges.

~~1.~~ $\int_0^3 \frac{1}{\sqrt{9-3x}} dx$

2. $\int_3^\infty \frac{1}{x^2-4x+7} dx$

3. $\int_e^\infty \frac{1}{x(\ln x)^3} dx$

4. $\int_0^\infty \frac{1}{(x+2)(2x+5)} dx$

5. $\int_7^\infty \frac{1}{x^2-8x+19} dx$

~~6.~~ $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$

7. $\int \frac{1}{(x+3)(3x+1)} dx$

8. $\int_2^\infty \frac{1}{x^2-2x+4} dx$

9. $\int \frac{1}{x^2+2x+2} dx$

~~10.~~ $\int_0^4 \frac{1}{(8-2x)^{\frac{1}{3}}} dx$

11. $\int_2^\infty \frac{1}{(x^2+4)^2} dx$

~~12.~~ $\int_{-1}^1 \frac{1}{\sqrt{1-x^2}} dx$

~~13.~~ $\int_0^1 \frac{1}{\sqrt{x}} dx$

~~14.~~ $\int_0^1 \frac{1}{x} dx$

15. $\int_1^\infty \frac{1}{x} dx$

~~16.~~ $\int_0^1 \frac{1}{x^2} dx$

Crossed out:

- improper integrals of "Type 2".
- rational functions requiring long division

(not covered on our midterm).

$$\cancel{17.} \int_0^{\frac{\pi}{2}} \tan x \, dx$$

$$\cancel{18.} \int_0^1 \frac{1-2x}{\sqrt{x-x^2}} \, dx$$

$$19. \int_0^{\infty} e^{-x} \, dx$$

$$\cancel{20.} \int_3^4 \frac{1}{(x-4)^2} \, dx$$

$$\cancel{21.} \int_1^2 \frac{1}{x \ln x} \, dx$$

$$\cancel{22.} \int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$$

$$23. \int_1^{\infty} \frac{\ln x}{x} \, dx$$

$$\cancel{24.} \int_0^{\infty} \frac{1}{x+x^2} \, dx$$

$$25. \int_{-\infty}^{\infty} \frac{x}{(x^2+4)^{\frac{3}{2}}} \, dx$$

$$\cancel{26.} \int_{-4}^4 \frac{1}{(x+4)^{\frac{2}{3}}} \, dx$$

$$27. \int_{-\infty}^{\infty} |x|e^{-x^2} \, dx$$

$$28. \int \frac{2x-5}{x^2+2x+2} \, dx$$

$$\cancel{29.} \int_0^1 \frac{1}{e^x - e^{-x}} \, dx$$

$$\cancel{30.} \int_0^1 \frac{e^x}{\sqrt{e^x-1}} \, dx$$

$$31. \int \frac{1}{x^2+4x+5} \, dx$$

$$\cancel{32.} \int_0^1 \ln x \, dx$$

$$\cancel{33.} \int_0^1 \frac{1}{(1-x^2)^{\frac{3}{2}}} \, dx$$

$$\cancel{34.} \int_1^5 \frac{x}{\sqrt{x-1}} \, dx$$

$$35. \int \frac{1}{x(x^2 + 1)} dx$$

$$36. \int_{-\infty}^{\infty} \frac{1}{x^2 - 6x + 10} dx$$

$$37. \int_0^{\infty} \frac{x}{e^x} dx$$

$$\cancel{38.} \int_{-5}^0 \frac{x}{x^2 + 4x - 5} dx$$

$$\cancel{39.} \int \frac{x^5 + 2}{x^2 - 1} dx$$

$$\cancel{40.} \int_0^6 \frac{1}{(x-2)^2} dx$$

$$41. \int_0^{\infty} \frac{1}{x^2 + 3x + 2} dx$$

$$\cancel{42.} \int_0^{\frac{\pi}{2}} \tan^2 x dx$$

$$\cancel{43.} \int_0^2 \frac{1}{(4-x^2)^{\frac{3}{2}}} dx$$

$$44. \int_{-\infty}^1 xe^{4x} dx$$

$$45. \int \frac{4x^2 + 7x + 6}{(x+2)(x^2+4)} dx$$

$$46. \int_1^{\infty} \frac{1}{x(x+1)} dx$$

$$\cancel{47.} \int_{-3}^{-2} \frac{1}{x^2 - 4} dx$$

$$\cancel{48.} \int_0^1 \arcsin x dx \text{ (leads to improper integral)}$$

$$\cancel{49.} \int \frac{2x^3}{x^2 + 3} dx$$

$$\cancel{50.} \int \frac{x^3 + 7x + 1}{x^2 + 1} dx$$

$$\cancel{51.} \int \frac{x^4 + x^3 + 2x^2 + 6x + 2}{(x+1)(x^2+1)} dx$$

Sequences: For each of the following sequences, decide whether it converges or diverges. If it converges, compute its limit.

52. $\left\{ \frac{1+n-7n^4}{3n^4+8n^3+9} \right\}_{n=1}^{\infty}$

53. $\left\{ \frac{n^3}{(n+1)^3} \right\}_{n=1}^{\infty}$

54. $\left\{ \left(\frac{n-5}{n} \right)^n \right\}_{n=1}^{\infty}$

~~55.~~ $\left\{ \frac{2^n}{n!} \right\}_{n=1}^{\infty}$

~~56.~~ $\left\{ \frac{n!}{3^n} \right\}_{n=1}^{\infty}$

57. $\left\{ \frac{(2n+3)!}{(2n+5)!} \right\}_{n=1}^{\infty}$

58. $\left\{ \arctan(n^2+1) \right\}_{n=1}^{\infty}$

59. $\left\{ \frac{\sqrt{n}}{(\ln n)^2} \right\}_{n=1}^{\infty}$

60. $\left\{ (e^n + n)^{\frac{1}{n}} \right\}_{n=1}^{\infty}$

61. $\left\{ n^{\frac{1}{n}} \right\}_{n=1}^{\infty}$

62. $\left\{ n \sin \left(\frac{1}{n} \right) \right\}_{n=1}^{\infty}$

crossed out:
problems requiring
the squeeze thm.

Series: Find the sum for each of the following series (all of which converge):

63. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$

64. $\sum_{n=0}^{\infty} \frac{1}{4^n} - \frac{1}{7^n}$

65. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{n-1}}{3^{n+1}}$

66. $\sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{4n-1}}$

crossed out:
"Telescoping" series

$$\cancel{67.} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$$

$$68. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3 \cdot 2^n}$$

$$\cancel{69.} \sum_{n=1}^{\infty} e^{\frac{1}{n}} - e^{\frac{1}{n+1}}$$

$$70. \sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}}$$

$$\cancel{71.} \sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$

$$72. \sum_{n=1}^{\infty} \frac{(-1)^{n4^n}}{9^{n-1}}$$

More Series: Determine whether each of the following series converge or diverge. Name any convergence test(s) you use, and justify that it's legal to use them:

$$73. \sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$$

$$74. \sum_{n=1}^{\infty} \frac{2n + \ln n}{n + 2010}$$

$$75. \sum_{n=1}^{\infty} \frac{e^n}{n^2}$$

$$76. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 1}$$

$$77. \sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$$

$$78. \sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

$$79. \sum_{n=1}^{\infty} \frac{n^2 + 1}{2n^2 \sqrt{n} + 9}$$

$$80. \sum_{n=1}^{\infty} \frac{\sqrt{n} + 3}{4n^2 - 2}$$

crossed out
some trickier uses of
the ratio test
(to be covered later)

$$81. \sum_{n=1}^{\infty} \frac{n^{19} + 40n^6 + 4n^3 + 19}{4 + 17n^5 + n^{20}}$$

$$82. \sum_{n=1}^{\infty} \frac{1}{n(\ln 2)^n}$$

$$83. \sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$$

~~$$84. \sum_{n=1}^{\infty} \frac{\ln n}{e^n}$$~~

$$85. \sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$86. \sum_{n=1}^{\infty} \frac{(-1)^n n}{3n + 2}$$

$$87. \sum_{n=1}^{\infty} n e^{-n^2}$$

$$88. \sum_{n=1}^{\infty} \frac{n!}{10^{4n}}$$

$$89. \sum_{n=1}^{\infty} e^{-2n}$$

$$90. \sum_{n=1}^{\infty} \frac{1 + 3n^3}{n^5}$$

$$91. \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^7}$$

$$92. \sum_{n=1}^{\infty} \frac{\arctan n}{1 + n^2}$$

$$93. \sum_{n=1}^{\infty} \frac{n^7}{e^n}$$

~~$$94. \sum_{n=1}^{\infty} \frac{n! \ln n}{n^2 3^n}$$~~

$$95. \sum_{n=1}^{\infty} \frac{2n + 5}{5n^3 + 3n^2}$$

$$96. \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$97. \sum_{n=1}^{\infty} \frac{1}{n+7}$$

$$98. \sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^2 + 1}$$

$$99. \sum_{n=1}^{\infty} \frac{7}{25 + n^2}$$

~~$$100. \sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$$~~

$$101. \sum_{n=1}^{\infty} \frac{n!}{(2n-1)!}$$

$$102. \sum_{n=1}^{\infty} 3 + \frac{1}{3^n}$$

~~$$103. \sum_{n=1}^{\infty} \frac{n!}{n^n}$$~~

$$104. \sum_{n=1}^{\infty} e^{\frac{1}{n}}$$

$$105. \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

$$106. \sum_{n=1}^{\infty} \frac{3}{n^3 7^n}$$

$$107. \sum_{n=1}^{\infty} \frac{2^n n^2}{(n+1)!}$$

$$108. \sum_{n=1}^{\infty} \frac{e^{2n} n!}{9^n}$$

$$109. \sum_{n=2}^{\infty} \frac{6}{n^6} + \frac{1}{(n+1)^6}$$

~~$$110. \sum_{n=1}^{\infty} \frac{(2n)^n n!}{(2n)!}$$~~

~~$$111. \sum_{n=1}^{\infty} \frac{4^n (n!)^3}{(2n)! n^n}$$~~

Even More Series: Determine whether each of the following series converges absolutely, converges conditionally, or diverges. Justify your answers.

$$112. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5n}$$

$$113. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$$

$$114. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{7n-3}$$

$$115. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{10n+1}$$

$$116. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$$

$$117. \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$$

$$118. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^{n^2}}$$

$$119. \sum_{n=2}^{\infty} \frac{n(-3)^{2n+1}}{10^n}$$

~~$$120. \sum_{n=1}^{\infty} \frac{7^n}{n^n}$$~~

~~$$121. \sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$$~~

$$122. \sum_{n=1}^{\infty} \frac{(-4)^{2n+1}}{n10^n}$$

$$123. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\arctan n}{n^7+n}$$

$$124. \sum_{n=1}^{\infty} \frac{(n+2)!}{3^n (n!)^2}$$

~~$$125. \sum_{n=1}^{\infty} \frac{(-1)^n (3n)! n^2}{8^n (n!)^2 n^n}$$~~

~~$$126. \sum_{n=1}^{\infty} \frac{(-1)^n (\ln n) \pi^n (2n)!}{n^n 4^n n!}$$~~