Math 121 Final Exam May 14, 2016

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, $e^{3\ln 3}$, $\arctan(\sqrt{3})$, or $\cosh(\ln 3)$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [18 Points] Evaluate the following **limit**. Please justify your answer. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to\infty}\left(\arcsin\left(\frac{1}{x}\right)+e^{\frac{1}{x}}\right)^{x}$ (b) $\displaystyle\lim_{x\to 0}\frac{xe^x-\arctan x}{\ln(1+3x)-3x}$

(c) Compute $\displaystyle\lim_{x\to 0}\frac{xe^x-\arctan x}{\ln(1+3x)-3x}$ **again** using series.

2. [18 Points] Evaluate each of the following **integrals**.

(a) $\displaystyle\int\frac{1}{(x^2+4)^2}\,dx$ (b) $\displaystyle\int_{-1}^{0}x^4\arcsin x\,dx$

3. [36 Points] For each of the following **improper integrals**, determine whether it converges or diverges. If it converges, find its value.

(a) $\displaystyle\int_0^1\sqrt{x}\,\ln x\,dx$ (d) $\displaystyle\int_1^{\sqrt{3}}\frac{x^4-x^3+3x^2-x+2}{x^3-x^2+3x-3}\,dx=\int_1^{\sqrt{3}}\frac{x^4-x^3+3x^2-x+2}{(x-1)(x^2+3)}\,dx$

(b) $\displaystyle\int_1^{\infty}\frac{e^{\frac{1}{x}}}{x^3}\,dx$ (c) $\displaystyle\int_{2\sqrt{3}}^{4}\frac{1}{\sqrt{16-x^2}}\,dx$ (e) $\displaystyle\int_7^{\infty}\frac{1}{x^2-8x+19}\,dx$

4. [18 Points] Find the **sum** of each of the following series (which do converge):

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\,5^{n+2}}{2^{3n-1}}$ (b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\,(\ln(27))^n}{3^{n+1}\,n!}$ (c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{2^{4n}\,(2n)!}$

(d) $-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\dots$ (e) $-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\dots$ (f) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{9^n\,(2n+1)!}$

5. [35 Points] In each case determine whether the given series is **absolutely convergent**, **conditionally convergent**, or **divergent**. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\,(n^5+7)}{n^7+5}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\arctan(\sqrt{3}\,n^2+1)}{n^2+\sqrt{3}}$

(c) $\displaystyle\sum_{n=1}^{\infty}\arctan\left(\frac{\sqrt{3}\,n^2+1}{n^2+\sqrt{3}}\right)$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\,n}{n^2+1}$ (e) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\,(\ln n)\,(3n)!}{n^n\,2^{4n}\,(n!)^2}$

6. [15 Points] Find the **Interval** and **Radius** of Convergence for the following power series $\sum_{n=1}^{\infty} \frac{(-1)^n\ (3x-5)^n}{n^8 \cdot 7^n}$. Analyze carefully and with full justification.

7. [20 Points] Consider the region bounded by $y = \arctan x$, $y = 0$, $x = 0$ and $x = 1$. Rotate the region about the y–axis.

(a) **Sketch** the resulting solid, along with one of the approximating cylindrical shells.

(b) **Set-up** the integral to compute the volume of this solid using the Cylindrical Shells Method.

(c) **Compute** your integral in part (b) above.

(d) Use MacLaurin Series to **Estimate** the integral in part (b) above with error less than $\frac{2\pi}{20}$. Justify.

8. [20 Points] Consider the Parametric Curve represented by $x = e^t + \frac{1}{1+e^t}$ and $y = 2\ln(1+e^t)$.

(a) Write the **equation of the tangent line** to this curve at the point where $t = 0$.

(b) **COMPUTE** the **arclength** of this parametric curve for $0 \le t \le \ln 3$.

9. [20 Points] For each of the following parts, do the following **two** things:

1. Sketch the Polar curves and shade the described bounded region.

2. Set-Up but **DO NOT EVALUATE** the Integral representing the area of the described bounded region.

(a) The **area** bounded outside the polar curve $r = 3 + 3\cos\theta$ and inside the polar curve $r = 9\cos\theta$.

(b) The **area** bounded outside the polar curve $r = 1$ and inside the polar curve $r = 2\sin\theta$.

(c) The **area** that lies inside both of the curves $r = 1 + \sin\theta$ and inside the polar curve $r = 1 - \sin\theta$.

(d) The **area** bounded outside the polar curve $r = 1$ and inside the polar curve $r = 2\sin(2\theta)$.