Integration by parts examples

(1) $\displaystyle\int_0^1 x\cdot e^{2x}\,dx$ $\qquad u=x \quad dv=e^{2x}dx$
$\qquad du=dx \quad v=\frac{1}{2}e^{2x}$

$$=\left[\tfrac{1}{2}xe^{2x}\right]_0^1-\int_0^1\tfrac{1}{2}e^{2x}dx = \left[\tfrac{1}{2}xe^{2x}\right]_0^1-\left[\tfrac{1}{4}e^{2x}\right]_0^1$$

$$=\tfrac{1}{2}e^2-0-\tfrac{1}{4}e^2+\tfrac{1}{4}$$

$$=\boxed{\tfrac{1}{4}e^2+\tfrac{1}{4}}$$

(2) $\displaystyle\int x^3e^{2x}dx$

using the tabular method:

x^3	$+$	e^{2x}
$3x^2$	$-$	$\frac{1}{2}e^{2x}$
$6x$	$+$	$\frac{1}{4}e^{2x}$
6	$-$	$\frac{1}{8}e^{2x}$
0	$+S$	$\frac{1}{16}e^{2x}$

$$\tfrac{1}{2}x^3e^{2x}-\tfrac{3}{4}x^2e^{2x}+\tfrac{6}{8}xe^{2x}-\tfrac{6}{16}e^{2x}+C$$

$$=\boxed{\tfrac{1}{2}x^3e^{2x}-\tfrac{3}{4}x^2e^{2x}+\tfrac{3}{4}xe^{2x}-\tfrac{3}{8}e^{2x}+C}$$

(3) $\displaystyle\int_1^e x\cdot \ln(x)\,dx$ $\qquad u=\ln x \quad dv=x\,dx$
$\qquad du=\frac{1}{x}dx \quad v=\frac{1}{2}x^2$

$$=\left[\tfrac{1}{2}x^2\ln(x)\right]_1^e-\int_1^e\tfrac{1}{2}x\,dx$$

$$=\tfrac{1}{2}e^2\cdot 1-\tfrac{1}{2}\cdot 1^2\cdot 0-\left[\tfrac{1}{4}x^2\right]_1^e$$

$$=\tfrac{1}{2}e^2-\tfrac{1}{4}e^2+\tfrac{1}{4}$$

$$=\boxed{\tfrac{1}{4}e^2+\tfrac{1}{4}}$$

④ $\int \arctan(x)\,dx$ $u = \arctan x$, $du = \frac{1}{1+x^2}\,dx$ $dv = dx$, $v = x$

$= x\cdot\arctan(x) - \int \frac{x}{1+x^2}\,dx$ $\left.\begin{array}{l} u = 1+x^2 \\ du = 2x\,dx \end{array}\right\}$ not the same "u" as before.

$= x\cdot\arctan(x) - \int \frac{1/2}{u}\,du$

$= \boxed{x\cdot\arctan(x) - \frac{1}{2}\ln|1+x^2| + C}$

⑤ $\int e^{-x}\cdot\cos(2x)\,dx$ $u = \cos(2x)$, $du = -2\sin(2x)$ $dv = e^{-x}dx$, $v = -e^{-x}$

$= -e^{-x}\cos(2x) - \int 2e^{-x}\sin(2x)\,dx$ $u = \sin(2x)$, $du = 2\cos(2x)$ $dv = 2e^{-x}dx$, $v = -2e^{-x}$

$= -e^{-x}\cos(2x) + 2e^{-x}\sin(2x) - \int 4\underline{e^{-x}\cos(2x)\,dx}$

solving gives

$$\int e^{-x}\cos(2x)\,dx = \boxed{-\frac{1}{5}e^{-x}\cos(2x) + \frac{2}{5}e^{-x}\sin(2x) + C}$$

⑥ $\int_1^{e^2} (\ln x)^2\,dx$ $u = (\ln x)^2$, $du = 2\ln x\cdot\frac{1}{x}$ $dv = dx$, $v = x$

$= \left[x\cdot(\ln x)^2\right]_1^{e^2} - \int_1^{e^2} 2\ln x\,dx$ $u = \ln x$, $du = 1/x\,dx$ $dv = 2dx$, $v = 2x$

$= e^2\cdot 2^2 - 1\cdot 0^2 - \left[2x\ln x\right]_1^{e^2} + \int_1^{e^2} (2x/x)\,dx$

$= 4e^2 - 2e^2\cdot 2 + 2\cdot 1\cdot 0 + \left[2x\right]_1^{e^2}$

$= \boxed{2e^2 - 2}$