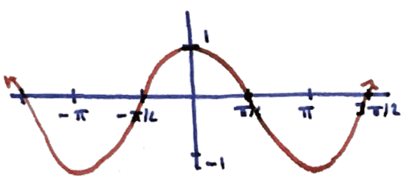
Inverse trig. practice

math121

① Define $\arccos(x)$ to be the angle ϑ in $[0, \pi]$ s.t. $\cos\vartheta = x$.

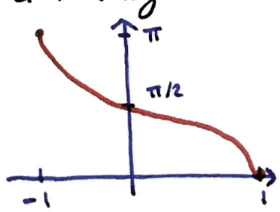
The following exercises are meant to allow you to apply the same sort of reasoning as we used to study $\arctan(x)$ & $\arcsin(x)$.

a) Why do we use $[0, \pi]$ instead of $[-\pi/2, \pi/2]$ (like with $\arcsin x$)?



cosx takes only positive values on $[-\pi/2, \pi/2]$, & it takes each one twice, so the choice of arccos(x) wouldn't be unique. On $[0, \pi]$, each value of cosx (from -1 to 1) occurs exactly once.

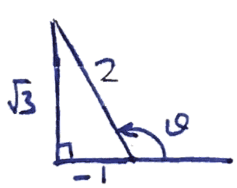
b) Sketch the graph of $\arccos(x)$. What is its domain & range?



Domain: $[-1, 1]$

Range: $[0, \pi]$

c) Evaluate $\sin(\arccos(-1/2))$.



$= \boxed{\sqrt{3}/2}$

$(\arccos(-1/2)) = 2\pi/3$, ie. $120°$).

d) Use implicit differentiation to find $\frac{d}{dx}(\arccos(x))$.

$y = \arccos(x)$

$\cos y = x$

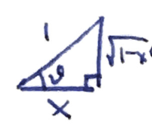
$-\sin(y) \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = -\frac{1}{\sin y}$

so

$\frac{d}{dx}(\arccos(x)) = -\frac{1}{\sin(\arccos(x))}$

$= \boxed{-\frac{1}{\sqrt{1-x^2}}}$



// comment: arcsinx is used more often in integrals. Try to guess why.

2) a) Find $\frac{d}{dx}\arcsin(\sqrt{x})$.

$$= \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}} \cdot \text{(chain rule)}$$

$$= \boxed{\frac{1}{2\sqrt{x}\cdot(1-x)}}$$

b) Find $\frac{d}{dx}\arctan\left(\frac{x}{2}\right)$.

$$= \frac{1}{1+(x/2)^2}\cdot\frac{1}{2}$$

$$= \frac{1}{2(1+x^2/4)} \quad \text{(chain rule)}$$

$$= \boxed{\frac{2}{4+x^2}}$$

3) a) Find $\int \frac{e^x}{1+e^{2x}}dx$ $\quad u=e^x$, $du=e^x dx$

$$= \int \frac{du}{1+u^2}$$

$$= \arctan(u)+C$$

$$= \boxed{\arctan(e^x)+C}$$

b) Find $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$ $\quad u=\cos x$, $du=-\sin x\,dx$

$$= \int_1^0 \frac{(-1)}{1+u^2}du$$

$$= -\left[\arctan(u)\right]_1^0$$

$$= \boxed{\pi/4}$$

4) a) Find $\int \frac{1}{25+x^2}dx$

$$= \frac{1}{25}\cdot\int\frac{1}{1+x^2/25}dx \quad u=x/5,\ du=\tfrac{1}{5}dx$$

$$\cdots \frac{1}{25}\int\frac{5du}{1+u^2}$$

$$= \frac{1}{5}\arctan(u)+C$$

$$= \boxed{\frac{1}{5}\arctan\left(\frac{x}{5}\right)+C}$$

b) Find $\int\frac{1}{\sqrt{9-x^2}}dx$

$$= \frac{1}{3}\int\frac{1}{\sqrt{1-x^2/9}}dx \quad u=x/3,\ du=\tfrac{1}{3}dx$$

$$= \frac{1}{3}\int\frac{3du}{\sqrt{1-u^2}}$$

$$= \arcsin(u)+C$$

$$= \boxed{\arcsin\left(\frac{x}{3}\right)+C}$$

// Hint: try to scale the numerator & denominator to get a more familiar form.