Review: e^x & some core concepts 1/22/18

// These problems are not to hand in, but they may help refresh
// concepts on the first homework.
// Work with those around you, and call me over for questions!

- Limits.

1) $\lim_{x\to(\pi/2)^-} e^{\tan x}$
$= \lim_{u\to\infty} e^u$ (since $\lim_{x\to(\pi/2)^-} \tan(x) = \infty$)
$= \boxed{\infty}$

2) $\lim_{x\to(-\pi/2)^+} e^{\tan x}$
$= \lim_{u\to-\infty} e^u$ (since $\lim_{u\to(-\pi/2)^+} \tan x = -\infty$)
$= \boxed{0}$

- Derivatives

3) $\frac{d}{dx}(e^{5x}) = e^{5x}\cdot\frac{d}{dx}(5x)$
$= \boxed{5e^{5x}}$

4) $\frac{d}{dx}\left(x^{5\cdot e}\right)$ (constant power of x)
$= \boxed{5e\cdot x^{5e-1}}$

5) $\frac{d}{dx}\left(\tan(e^x)\right) = \sec^2(e^x)\cdot\frac{d}{dx}(e^x)$
$= \boxed{e^x\cdot\sec^2(e^x)}$

6) $\frac{d}{dx}\left(e^{\tan(e^x)}\right) = e^{\tan(e^x)}\cdot\frac{d}{dx}(\tan(e^x))$
$= \boxed{e^{\tan(e^x)}\cdot\sec^2(e^x)\cdot e^x}$

- Integrals & u-substitution

7) $\int_0^L e^{-2x}\,dx$ $u=-2x$, $du=-2dx$ (express the answer in terms of L)
$= \int_0^{-2L}(-\tfrac{1}{2})e^u\,du = \left[-\tfrac{1}{2}e^u\right]_0^{-2L} = -\tfrac{1}{2}e^{-2L}+\tfrac{1}{2}e^0 = \boxed{-\tfrac{1}{2}e^{-2L}+\tfrac{1}{2}}$

8) $\int e^x(e^x+1)^3\,dx$ $u=e^x+1$, $du=e^x dx$
$= \int u^3\,du = \tfrac{1}{4}u^4+C = \boxed{\tfrac{1}{4}(e^x+1)^4+C}$

9) $\int_0^1 e^{2x}\cdot\sqrt{1+e^x}\,dx$ $u=1+e^x$, $du=e^x dx$ ie. $dx=\frac{1}{e^x}du$
$= \int_{x=0}^{x=1} e^{2x}\sqrt{u}\cdot\frac{1}{e^x}du = \int_{x=0}^{x=1} e^x\sqrt{u}\,du = \int_2^{1+e}(u-1)\sqrt{u}\,du = \int_2^{1+e}(u^{3/2}-u^{1/2})du$
$= \left[\tfrac{2}{5}u^{5/2}-\tfrac{2}{3}u^{3/2}\right]_2^{1+e} = \boxed{\tfrac{2}{5}(1+e)^{5/2}-\tfrac{2}{3}(1+e)^{3/2}-\tfrac{2}{5}2^{5/2}+\tfrac{2}{3}2^{3/2}}$