Some curve sketches

math 111, 10/31/17.

$$\boxed{f(x) = \frac{x^2}{\sqrt{x^4+4}}}$$

Derivatives (work omitted): $f'(x) = \dfrac{8x}{(x^2+4)^{3/2}}$

$$f''(x) = -8 \cdot \frac{5x^4-4}{(x^4+4)^{5/2}}$$

- Domain: $\underline{(-\infty, \infty)}$ (denom. is never 0).
- Intercepts: $(0,0)$ is the only x- or y-intercept.
- symmetry: $f(x) = f(-x)$; the function is <u>even</u>.
- Asymptotes: no vertical asymptotes (no discontinuities).

  To find horiz. asymptotes:

$$\lim_{x\to-\infty} \frac{x^2}{\sqrt{x^4+4}} = \lim_{x\to-\infty} \frac{1}{\frac{1}{x^2}\sqrt{x^4+4}} = \lim_{x\to-\infty} \frac{1}{\sqrt{1+4/x^4}}$$

$$= 1/\sqrt{1+4/\infty} = \underline{1}.$$

similarly $\displaystyle\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+4}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+4/x^4}} = 1$

horiz. asymptote at $\underline{y=1}$

- Sign chart for $f'(x)$:

	$8x$	$\frac{1}{(x^2+4)^{3/2}}$	f'	f is
$x<0$	$-$	$+$	$-$	dec.
$x>0$	$+$	$+$	$+$	inc.

} local min. @ $x=0$. $(0,0)$.

$$\boxed{f(x) = \frac{x^2}{\sqrt{x^4+4}}, \text{ cont.}}$$

- sign chart for $f''(x)$: $\quad f''(x)=0$ when $5x^4=4$, ie $x = \pm\sqrt[4]{4/5}$.

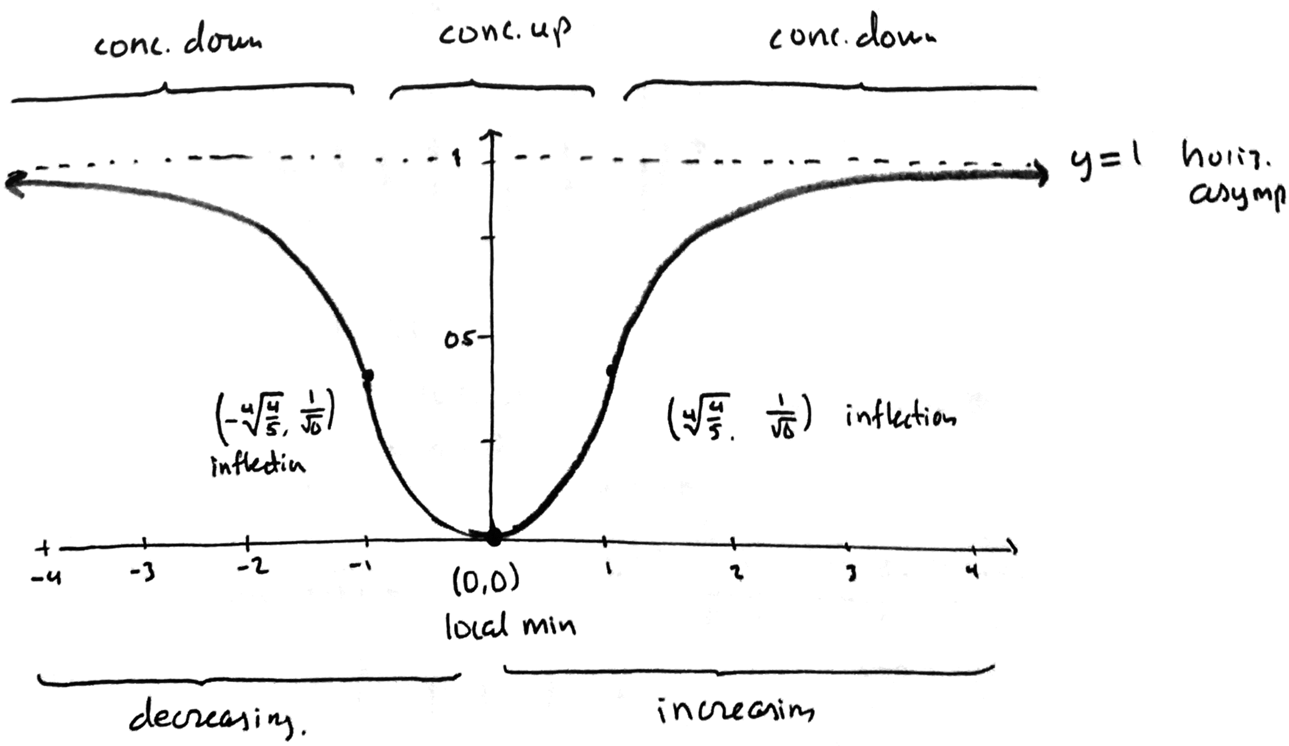
	$5x^4-4$	$\cdot \frac{-8}{(x^4+4)^{5/2}}$	f''	conc. of f
$x < -\sqrt[4]{4/5}$	+	−	−	down
$-\sqrt[4]{4/5} < x < \sqrt[4]{4/5}$	−	−	+	up
$x > \sqrt[4]{4/5}$	+	−	−	down

$\Rightarrow$ inflection points at $x = \pm\sqrt[4]{4/5}$ $(\simeq \pm 0.945)$

Note $f(\pm\sqrt[4]{4/5}) = \frac{\sqrt{4/5}}{\sqrt{\frac{4}{5}+4}} = \frac{\sqrt{4/5}}{\sqrt{24/5}} = \frac{1}{\sqrt{6}}$. $(\simeq 0.408)$

so inflection pts. at $\left(\pm\sqrt[4]{\frac{4}{5}}, \frac{1}{\sqrt{6}}\right)$.

- The sketch:



$$\boxed{f(x) = \frac{x^2}{1-x^2}} \qquad f'(x) = \frac{2x}{(1-x^2)^2} \qquad f''(x) = -2\,\frac{1+3x^2}{(x^2-1)^3}.$$

- Domain : everything but $x = \pm 1$ (where denom. is 0).
- Intercepts: just $(0,0)$.
- Symmetry : this is an <u>even</u> function, since $\frac{(-x)^2}{1-(-x)^2} = \frac{x^2}{1-x^2}$.
- Asymptotes: vertical asymptotes at $\underline{x=\pm 1}$, since the numerator goes to $(\pm 1)^2 = 1$ but the denominator goes to 0.

Horiz. asymptotes:

$$\lim_{x\to\pm\infty} \frac{x^2}{1-x^2} = \lim_{x\to\pm\infty} \frac{1}{1-1/x^2} = \frac{1}{1-1/(\pm\infty)^2}$$

$$= \frac{1}{1-1/\infty} = \underline{1}.$$

$\underline{y=1}$ is a horiz. asy. (on both sides).

- Sign chart for $f'(x)$:

divide at $x = 0, \pm 1$ ($f'(x) = 0$ at $x=0$, & $f'(x)$ undefined at $x = \pm 1$).

	$2x$	$\frac{1}{(1-x^2)^2}$	f'	f is...
$(-\infty,-1)$	$-$	$+$	$-$	dec.
$(-1,0)$	$-$	$+$	$-$	dec.
$(0,1)$	$+$	$+$	$+$	inc.
$(1,\infty)$	$+$	$+$	$+$	inc.

} local min. @ $(0,0)$

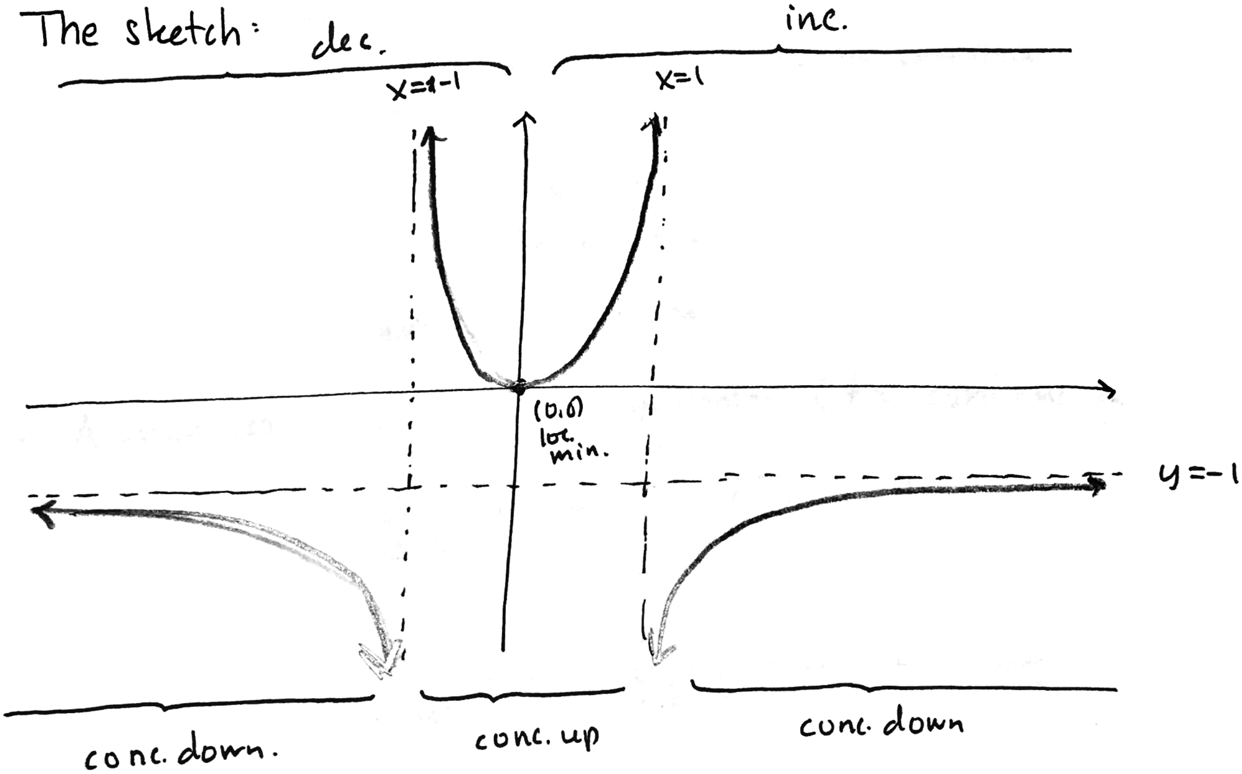
- Sign chart for $f''(x)$:

$f''(x)$ is never 0, since the numerator $1+3x^2$ is never 0.
$f''(x)$ is undefined at $x = \pm 1$. So divide there.

	-2	$1+3x^2$	$\frac{1}{(x^2-1)^3}$	f''	f concave...
$(-\infty,-1)$	$-$	$+$	$+$	$-$	down
$(-1,1)$	$-$	$+$	$-$	$+$	up
$(1,\infty)$	$-$	$+$	$+$	$-$	down

$$\boxed{f(x) = \frac{x^2}{1-x^2}\text{, cont.}}$$

- The sketch:



$$\boxed{f(x) = \sqrt{x^2+1} - x}$$

$f'(x) = \frac{x}{\sqrt{x^2+1}} - 1$ $\quad f''(x) = \frac{1}{(x^2+1)^{3/2}}$

(work omitted)

- Domain: $(-\infty, \infty)$
- Intercepts: $(0, 1)$ is y-intercept.

  no x-intercept since $x \neq \sqrt{x^2+1}$
- Symmetry: not even, odd, or periodic.
- Asymptotes: no vertical asymptotes ($f(x)$ continuous).

  horiz: $\lim\limits_{x \to -\infty} f(x) = \sqrt{(-\infty)^2+1} - (-\infty)$

  $= \infty + \infty = \infty.$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(\sqrt{x^2+1} - x\right) = \lim_{x \to \infty} \frac{(\sqrt{x^2+1} - x)(\sqrt{x^2+1} + x)}{\sqrt{x^2+1} + x}$$

$$= \lim_{x \to \infty} \frac{(x^2+1) - x^2}{\sqrt{x^2+1} + x} = \lim_{x \to \infty} \frac{1}{\sqrt{x^2+1} + x}$$

$$= \frac{1}{\sqrt{\infty^2+1} + \infty} = \frac{1}{\infty + \infty} = 0.$$

So $\underline{y=0}$ is a horiz. asymptote. (approached on the right only).

- Sign chart of $f'(x)$:

solving $f'(x)=0$ gives

$$1 = \frac{x}{\sqrt{x^2+1}}$$

$$\sqrt{x^2+1} = x$$

$$x^2+1 = x^2$$

$$1 = 0.$$

$\Rightarrow$ there are no sol'ns to $0 = f'(x)$.

$f'(x)$ is defined for all x

$\Rightarrow$ no critical points.

Now, $f'(0) = -1$.

Since $f'(x)$ never changes sign (no critical pts), it is always negative.

$\underline{f(x) \text{ decreases everywhere}}$.

$\boxed{f(x) = \sqrt{x^2+1} - x, \text{ cont.}}$

- Signs of $f''(x)$.

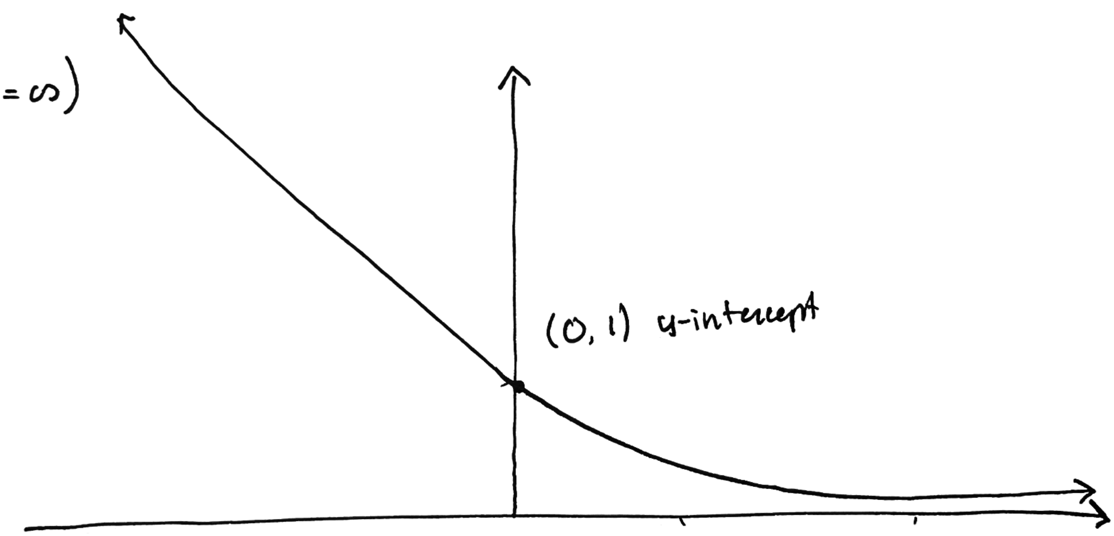
$f''(x) = \frac{1}{(x^2+1)^{3/2}}$ is always positive.

$(x^2+1>0)$.

So $f(x)$ is always concave up.

- Sketch:

$\left(\lim_{x\to-\infty} f(x) = \infty\right)$



y = 0 horiz asymptote approached on right.

always decreasing.
always concave up.