

More related rates problems

1. The pressure P , volume V , and temperature T of a balloon full of gas obeys the ideal gas law, $P \cdot V = C \cdot T$, where C is a constant and T is measured in Kelvin. At the current instant, the pressure is 100,000 Pascals, the volume is 1 m^3 , and the temperature is 300 Kelvin (that is, 27°C). The volume is expanding by $0.1 \text{ m}^3/\text{sec}$, and the temperature is increasing by $3 \text{ K}/\text{sec}$ (equivalently, $3^\circ\text{C}/\text{sec}$). Is the pressure of the gas in the balloon increasing or decreasing, and how quickly?
2. An oil spill occurs at sea. The oil gushes out from an offshore derrick and forms a circle whose area increases at a rate of $100 \text{ ft}^2/\text{min}$. How fast is the radius of the spill increasing when the spill is 20 ft across?
3. The sides of a rectangle change with respect to time. The width is increasing at a rate of $2 \text{ in}/\text{sec}$, while the length is decreasing at a rate of $3 \text{ in}/\text{sec}$. How fast is the area of the rectangle changing when the width is 6 in and the length is 8 in?
4. Gravel is dumped at a rate of $30 \text{ ft}^3/\text{min}$ onto a conical pile where the diameter of the base always equals twice the height. How fast is the height increasing when the pile is 10 ft high? Note that the volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.
5. A 10 ft ladder slides down a wall, with the top of the ladder moving at a rate of $3 \text{ ft}/\text{sec}$. How fast is the *angle* ϑ between the ladder and the ground decreasing when the top of the ladder is 6 ft from the ground?

A solution strategy for related-rates problems (see the textbook for a slightly different strategy)

1. Read the problem carefully.
2. Draw a figure depicting the situation (or two figures, one for “generic time” and one for the instant in question).
3. Identify all changing quantities; give them names and label them in the figure(s).
4. Write the information you are given in terms of values of these quantities and their derivatives.
5. Relate the quantities by an equation or equations (static relationship).
6. Differentiate both sides of your equation(s) to relate the quantities and their rates (dynamic relationship).
7. Substitute the known information into the equation(s), and solve for the desired information.