More related rates problems

1. The pressure P, volume V, and temperature T of a balloon full of gas obeys the ideal gas law, $P \cdot V = C \cdot T$, where C is a constant and T is measured in Kelvin. At the current instant, the pressure is 100,000 Pascals, the volume is 1 m^3, and the temperature is 300 Kelvin (that is, 27°C). The volume is expanding by 0.1 m^3/sec, and the temperature is increasing by 3 K/sec (equivalently, 3°C/sec). Is the pressure of the gas in the balloon increasing or decreasing, and how quickly?

$P(0) = 100{,}000 \quad P'(0) = ?$
$V(0) = 1 \quad V'(0) = 0.1$
$T(0) = 300 \quad T'(0) = 3$

$PV = C \cdot T$
$\Rightarrow P'V + PV' = CT'$
at $t=0$: $P'(0) \cdot 1 + 100000 \cdot 0.1 = C \cdot 3$
$\Rightarrow P'(0) = 3C - 10{,}000.$

solve for C:
$C = P(0)V(0)/T(0)$
$= 100{,}000/300 = 1000/3$
So:
$P'(0) = 1{,}000 - 10{,}000 = \boxed{-9000}$ (decreasing). (Pa/sec)

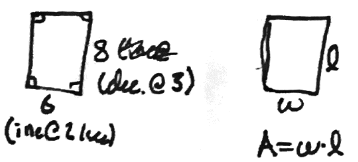
2. An oil spill occurs at sea. The oil gushes out from an offshore derrick and forms a circle whose area increases at a rate of 100 ft^2/min. How fast is the radius of the spill increasing when the spill is 20 ft across?

20 ft across $\Rightarrow$ radius 10.

at this moment:
$A(0) = 100\pi \quad A'(0) = 100$
$r(0) = 10$
want: $r'(0)$.

$A = \pi r^2$
$\Rightarrow A' = 2\pi r \cdot r'$
$\Rightarrow r'(0) = A'(0)/2\pi r(0)$
$= 100/2\pi \cdot 10 = \boxed{\frac{5}{\pi}}$ (ft/sec)

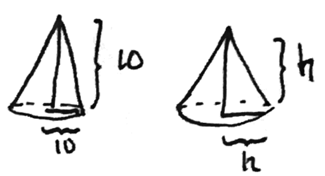
3. The sides of a rectangle change with respect to time. The width is increasing at a rate of 2 in/sec, while the length is decreasing at a rate of 3 in/sec. How fast is the area of the rectangle changing when the width is 6 in and the length is 8 in?



$A = w \cdot l$
$\Rightarrow A' = w'l + wl'$

At this moment, $w'(0) = 2$, $l(0) = 8$, $w(0) = 6$, $l'(0) = \cancel{3} -3$.
So $A'(0) = 2 \cdot 8 - 3 \cdot 6$
$= \boxed{-2}$ (ft^2/sec)

4. Gravel is dumped at a rate of 30 ft^3/min onto a conical pile where the diameter of the base always equals twice the height. How fast is the height increasing when the pile is 10 ft high? Note that the volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.

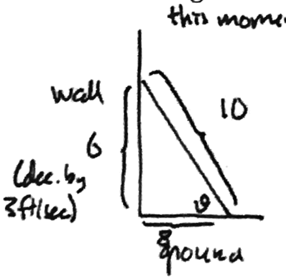


radius = height. So
$V = \frac{1}{3}\pi h^3.$
$\Rightarrow V' = \pi h^2 \cdot h'.$

at this moment, we know that
$V'(0) = 30$ & $h(0) = 10$.
So $30 = \pi \cdot 10^2 \cdot h'(0)$
$\Rightarrow h'(0) = \frac{30}{100\pi} = \boxed{\frac{3}{10\pi}}$ (ft/sec)

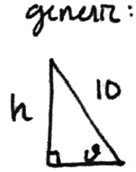
5. A 10 ft ladder slides down a wall, with the top of the ladder moving at a rate of 3 ft/sec. How fast is the *angle* ϑ between the ladder and the ground decreasing when the top of the ladder is 6 ft from the ground?

this moment:



(can compute 8 as $\sqrt{10^2 - 6^2}$)

general:



know:
$h(0) = 6$
$h'(0) = -3$
want:
$\vartheta'(0)$.

relation:
$\sin\vartheta = h/10$
$\Rightarrow (\cos\vartheta) \cdot \vartheta' = h'/10$
$\Rightarrow \vartheta'(0) = \frac{h'(0)}{10\cos\vartheta(0)}.$
From the "specific" picture, we deduce that at time 0, $\cos\vartheta = 8/10 = 4/5$.

Therefore:
$\vartheta'(0) = \frac{-3}{10 \cdot (8/10)}$
$= \boxed{-3/8}$
(rad/sec).
($\approx 21.5°$/sec).