math 111, 9/15/17

$\boxed{\text{Evaluating limits: examples.}}$ // for practice in class; not to hand in.

Evaluate each limit, or explain why it doesn't exist.

① $\lim\limits_{x\to 7} \frac{x-7}{|7-x|}$  Note: $|7-x| = \begin{cases} 7-x & x\le 7 \\ x-7 & x\ge 7. \end{cases}$

Hence $\lim\limits_{x\to 7^-} \frac{x-7}{|7-x|} = \lim\limits_{x\to 7^-} \frac{x-7}{7-x} = \lim\limits_{x\to 7^-} (-1) = -1$

and $\lim\limits_{x\to 7^+} \frac{x-7}{|7-x|} = \lim\limits_{x\to 7^+} \frac{x-7}{x-7} = \lim\limits_{x\to 7^+} (1) = 1.$

So the limit $\boxed{\text{does not exist}}$ (one-side limits not equal).

② $\lim\limits_{x\to 2} \frac{x^2-6x+8}{x-2} = \lim\limits_{x\to 2} \frac{\cancel{(x-2)}(x-4)}{\cancel{(x-2)}} = \lim\limits_{x\to 2} (x-4) = \boxed{-2}.$

③ $\lim\limits_{x\to 1} \frac{x^2+2x-3}{x^2-4x+3} = \lim\limits_{x\to 1} \frac{(x-1)(x+3)}{(x-1)(x-3)} = \lim\limits_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}} \cdot \lim\limits_{x\to 1} \frac{(x+3)}{(x-3)}$

$= \lim\limits_{x\to 1} (1)\left(\frac{1+3}{1-3}\right) = 1\cdot\frac{4}{-2} = \boxed{-2}$

④ $\lim\limits_{x\to 7} \frac{x^2-10x+21}{|7-x|}$ ← From either side: $\lim\limits_{x\to 7^\pm} \frac{x^2-10x+21}{|7-x|} = \lim\limits_{x\to 7^\pm} (x-3)\cdot \lim\limits_{x\to 7^\pm} \frac{x-7}{|7-x|}$

and now we can use #1 to conclude that $\lim\limits_{x\to 7^-} \frac{x^2-10x+21}{|7-x|} = (7-3)\cdot(-1) = \underline{-4}$

and $\lim\limits_{x\to 7^+} \frac{x^2-10x+21}{|7-x|} = (7-3)\cdot(+1) = \underline{4}$

So the limit $\boxed{\text{does not exist.}}$

⑤ $\lim\limits_{t\to 0} \frac{1-\sqrt{1+t}}{t^2+t} = \lim\limits_{t\to 0} \frac{1}{t+1} \cdot \lim\limits_{t\to 0} \frac{1-\sqrt{1+t}}{t} = 1\cdot \lim\limits_{t\to 0} \frac{1-\sqrt{1+t}}{t}$ . (if this exists)

Rationalize the numerator: $\frac{1-\sqrt{1+t}}{t} = \frac{1-\sqrt{1+t}}{t}\cdot\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}} = \frac{1-(1+t)}{t(1+\sqrt{1+t})} = -\frac{t}{t(1+\sqrt{1+t})} = -\frac{1}{1+\sqrt{1+t}}$ (for all $t\neq 0$).

Hence $\lim\limits_{t\to 0} \frac{1-\sqrt{1+t}}{t} = \lim\limits_{t\to 0} \frac{-1}{1+\sqrt{1+t}} = \frac{-1}{1+\sqrt{1+0}} = \boxed{-1/2}$