Worksheet 2 Solutions:

1. \( g(x) = \frac{x}{x-1} - \frac{x+2}{x} = \frac{a(x)}{x(x-1)} - \frac{b(x)}{x(x-1)} = \frac{a(x) - b(x)}{x(x-1)} \)

\( \Rightarrow \frac{x(x) - (x-1)(x+2)}{x(x-1)} = \frac{x^2 - (x-1)(x+2)}{x(x-1)} = \frac{x^2 - x^2 - 2x + 2}{x(x-1)} = \frac{-2x + 2}{x(x-1)} = \frac{-2}{x-2} \)

\( = \frac{x^2 - x}{x-2} \)

2. \( f \circ g(x) = f(g(x)) \)

(a) Note that the domain of a function is everything the function can take in. So, the domain of \( f(g(x)) \) will be defined by \( g(x) \). That's what goes in. Since \( g(x) \) is a value, specifically any value given by plugging in \( x \), the domain of \( f(g(x)) \) is the range of \( g(x) \).

(b) \( f \circ g = f(g(x)) = \frac{1}{\sqrt{x+2}} + 4 \)

\( \Rightarrow g \circ f = g(f(x)) = g(\sqrt{x+4}) = \sqrt{x+4} + 2 \)

\( f \circ g \) does NOT equal \( g \circ f \). Consider \( x = 0 \):

\( f(g(0)) = \frac{1}{\sqrt{0+2}} + 4 = \frac{1}{2} + 4 = \frac{9}{2} \)

\( g(f(0)) = \sqrt{0+4} + 2 = \sqrt{4} + 2 = 2 + 2 = 4 \)

These are not equal!

3. \( f(f(x)) = \frac{f(x) + 1}{f(x) - 1} \)

\( \Rightarrow \) You can break this up! \( f(x) = \frac{2+1}{2-1} = 3 \)

Remember to plug all of this in first.

\( \Rightarrow \) So \( \frac{f(2) + 1}{f(2) - 1} = \frac{3+1}{3-1} = \frac{4}{2} = 2 \)

\( f(2) + 1 = \frac{x+1}{x-1} + 1 = \frac{x+1 + x-1}{x-1} = \frac{2x}{x-1} \)

\( \Rightarrow \frac{x+1}{x-1} - 1 = \frac{x+1 - (x-1)}{x-1} = \frac{2}{x-1} \)

\( = \frac{2x}{x-1} \cdot \frac{x-1}{2} = \frac{2x}{2} = x \)
4. \( f(x) = \frac{1}{x+1} \Rightarrow f(x+h) - f(x) = \frac{1}{x+h+1} - \frac{1}{x+1} = \frac{x+1}{(x+1)(x+h+1)} - \frac{1}{x+1} \cdot \frac{1}{h} = \frac{-1}{(x+1)(x+h+1)} \)

5. \( f(x) = \frac{x-7}{x+3} \Rightarrow f(x+h) - f(x) = \frac{x+h-7}{x+h+3} - \frac{x-7}{x+3} = \frac{(x+3)(x+h-7)}{(x+3)(x+h+3)} - \frac{(x-7)(x+h+3)}{(x-7)(x+h+3)} \)

\[= \frac{x^2 + xh - 7x + 3x + 3h - 21}{(x+3)(x+h+3)} - \frac{x^2 + xh + 3x - 7x - 7h - 21}{(x+3)(x+h+3)} \]

\[= \frac{3h + 7h}{(x+3)(x+h+3)} = \frac{10h}{(x+3)(x+h+3)} \cdot \frac{1}{h} = \frac{10}{(x+3)(x+h+3)} \]

6. (a) \( \frac{\chi^2 - 8\chi - 8}{\chi^2 - 4} = \frac{(\chi - 2)(\chi + 4)}{(\chi - 2)(\chi + 2)} = \frac{\chi + 4}{\chi + 2} \)

(b) \( \frac{\chi^2 - 6\chi + 8}{\chi^2 - 5\chi - 14} = \frac{(\chi - 1)(\chi + 4)}{(\chi - 7)(\chi + 2)} = \frac{\chi + 4}{\chi - 7} \)

(c) \( \frac{\chi^2 - 6\chi + 8}{\chi^2 - 4} = \frac{(\chi - 2)(\chi + 4)}{(\chi - 2)(\chi + 1)} = \frac{\chi - 4}{\chi + 1} \)

(d) \( \frac{1}{t+1} - \frac{1}{t} = \frac{1}{t+1} - \frac{1}{t+1} = \frac{1}{t+1} - \frac{1}{t+1} = \frac{1}{t+1} \)

(e) \( \frac{t-1}{g(t^2) - 3} = \frac{t-1}{2t^2 + 1 - 3} = \frac{t-1}{2t^2 - 2} = \frac{t-1}{2(t^2 - 1)} = \frac{t-1}{2(t-1)(t+1)} = \frac{1}{2(t+1)} \)

(f) \( \frac{x^2 - 13x + 42}{x^2 - 4x + 12} \) can't simplify. Check: \( \frac{4 \pm \sqrt{16 - 4(12)}}{2} \) anymore?

This tells us there are no real solutions to this, so it can't be simplified more!
(g) \[ \frac{1}{x} - \frac{1}{|x|} \Rightarrow \text{Case 1: } |x|, \text{ } x \text{ is pos.} \]
\[ \frac{1}{x} - \frac{1}{x} = 0 = [0] \]
\[ \text{Case 2: } |x|, \text{ } x \text{ is neg.} \]
\[ \frac{1}{x} - \left( \frac{1}{-x} \right) = \frac{1}{x} + \frac{1}{x} = \frac{2}{x} \]

(h) \[ \frac{x+4}{x+4} \Rightarrow \text{Case 1: } |x+4|, \text{ } x+4 \text{ is pos.} \]
\[ \frac{x+4}{x+4} = [1] \]
\[ \text{Case 2: } |x+4|, \text{ } x+4 \text{ is neg.} \]
\[ \frac{x-4}{x+4} = [-1] \]

(i) \[ f(x) = \frac{1}{x}, \quad \frac{f(t-1)-2f(t)}{t^2-4} = \frac{1}{t-1} - 2 \left( \frac{1}{t} \right) = \frac{t - 2(t-1)}{t^2-4} = \frac{t^2-4}{t^2-4} = \frac{t+2}{t(t-1)} = \frac{-(t-2)}{t+2} \]

7. \[ f(g(x)) = \frac{x^3+1}{x^3+2} \]
Note: For problems like this one, try to find a repeating function that would "be plugged in":
\[ g(x) = x^3 \text{ and } f(x) = \frac{x+1}{x+2} \Rightarrow f(g(x)) = \frac{x^3+1}{x^3+2} \text{ and } g(f(l(x)) = \left( \frac{x+1}{x+2} \right)^3 \]

B. (a) Consider: \[ \frac{x+y}{x^2} = \frac{x(x+1)}{x^2} = \frac{x+1}{x} \] This is correct b/c the x can be factored out and cancelled.

\[ \frac{x+y^2}{x^2} \neq \frac{x+y^2}{z} \]
This is incorrect b/c we can't factor out an x in x\(y^2\). So we cancel the x in the denominator.

(b) Consider \[ \sqrt{x^2+y^4} = \sqrt{(xy)^2} = xy^2 \] This is correct b/c there are 2 copies of \(y^2\) in \(x^2y^4\), so we can simplify the radical.

\[ \text{Note: } \sqrt{a^n} = \sqrt{(a^n)^2} = a^n \]

Consider \[ \sqrt{x^2+y^4} \neq x+y^4 \]
The mistake made here was simplifying each individual term. In order for this to be true, we need \((x+y^2)^2 = x^2+y^4\) (what's under the radical).
\[ 4(x+y^2)(y+y^2) = x^2+2y^2+y^4 = x^2+y^4 \]

\[ \text{Note: } \sqrt{a^2+b^2} \neq \sqrt{a^2} + \sqrt{b^2} \]