Name: **Solutions**

- Keep phones off and out sight.
- No calculators, notes, books, or other aids.
- Do not talk during the quiz.
- Show all work.

1. You are watching a rocket launch from a spot 12 kilometers away from the launchpad. The rocket flies straight up after launching. Shortly after launch, you use a radar instrument to determine that the rocket is 15 kilometers away from your location, and that the distance between you and the rocket is growing by \( \frac{3}{10} \) of a kilometer per second. How quickly is the rocket rising at that instant?

   It may be useful to know that \( \sqrt{15^2 - 12^2} = 9 \).

   ![Diagram of rocket launch](image)

   \[ 12^2 + y^2 = h^2 \]

   \[ \text{Given: at key moment, } h = 15, \quad h' = \frac{3}{10} \]

   \[ \text{Want (at key moment): } y' = ? \]

   \[ y = \sqrt{h^2 - 12^2} = \sqrt{15^2 - 12^2} = 9 \]

   \[ \frac{d}{dt}(12^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(h^2) \]

   \[ 0 + 2y\cdot y' = 2h\cdot h' \]

   \[ 2\cdot 9\cdot y' = 2\cdot 15\cdot \frac{3}{10} \]

   \[ \Rightarrow y' = \frac{8\cdot 15 \cdot 3}{8\cdot 9\cdot 10} = \frac{45}{90} = \frac{1}{2} \]

   Rising at \( \frac{1}{2} \) of a km. per second.

(Question 2 is on the back)
2. A spherical balloon is being inflated by a pump. Its volume is increasing by $300 \text{ cm}^3$ per second. How quickly is the radius of the balloon increasing (in cm per second) when the radius is 20 cm?

(The volume of a sphere of radius $r$ is $\frac{4}{3} \pi r^3$.)

\[ V = \text{ volume of balloon} \]
\[ r = \text{ radius of balloon} \]

\[ V = \frac{4}{3} \pi r^3 \]

\text{Given info: at key moment, want (at key moment)}
\[ V' = 40 \]
\[ r = 20 \]

\text{Differentiate}
\[ V' = \frac{4}{3} \pi \frac{d}{dt}(r^3) = \frac{4}{3} \pi \cdot 3r^2 \cdot r' \]

\[ = 4 \pi r^2 \cdot r' \]

\text{Substitute & Solve (at key moment)}
\[ 400 = 4 \pi \cdot 20^2 \cdot r' \]
\[ = 4 \pi \cdot 400 \cdot r' \]
\[ = 1600 \pi \cdot r' \]

\[ \Rightarrow r' = \frac{400}{1600 \pi} = \frac{1}{4 \pi} \]

\text{Radius grows at $\frac{1}{4 \pi}$ cm/sec.}