Math 105  Test #1  Fall 2015

Name:

Read This First!

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted. Cell phones out of sight.

• Please read each question carefully. Show ALL work clearly in the space provided. You may use the backs of pages for additional work space.

• In order to receive full credit on a problem, solution methods must be complete, logical and understandable

• Answers must be clearly labeled in the spaces provided after each question.

• The exam consists of Questions 1–6, which total to 100 points. Question 7 is a bonus question (5 points extra credit) that is optional.

Grading - For Instructor Use Only

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1. [24 points] Compute the following limits. If $+\infty$ or $-\infty$ is a correct answer, please give it.

(a) $\lim_{x \to 1} \frac{1 + x^2}{1 + x}$

(b) $\lim_{x \to 1} \frac{x - 1}{x^2 + x - 2}$

(c) $\lim_{x \to 4^+} \frac{x^2 - 1}{4 - x}$

(d) $\lim_{x \to 1} \frac{\sqrt{x^2 + 1} - \sqrt{2}}{x - 1}$
2. [18 points] Consider the following graph:

(a) What is the domain of $f$? Express your answer in interval notation.

(b) For which $x$'s is $f(x) = 0$?

(c) For which $x$'s is $f(x) < 0$? Express your answer in interval notation.

(d) Is $f$ continuous at 2? Explain your answer using the definition of continuity.
3. [20 points] Consider the function defined by

\[
g(x) = \begin{cases} 
\frac{1}{x} & x > 0 \\
1 & x = 0 \\
2 - x^2 & x < 0.
\end{cases}
\]

(a) Draw the graph of \( g \).

(b) Use the graph of part (a) to find \( \lim_{x \to 0^+} g(x) \), \( \lim_{x \to 0^-} g(x) \), \( \lim_{x \to 0^+} g(x) \) and \( g(0) \).
4. [15 points] Let

\[ f(x) = \frac{x+1}{x+2} \quad \text{and} \quad g(x) = \frac{1-x}{1+x}. \]

Simplify \( f(g(x)) \) as much as possible.
5. [15 points] Suppose we know the limits
\[ \lim_{x \to 2} f(x) = 4, \quad \lim_{x \to 2} g(x) = 3, \quad \lim_{x \to 2} h(x) = 0. \]

(a) What do the limit laws say about \( \lim_{x \to 2} \frac{f(x)}{g(x)} \)?

(b) What do the limit laws say about \( \lim_{x \to 2} \frac{h(x)}{g(x)} \)?

(c) What do the limit laws say about \( \lim_{x \to 2} \frac{g(x)}{h(x)} \)?

6. [8 points] Find the equation of the line perpendicular to the line \( 2x + 5y = 10 \) that goes through the point \((-\frac{1}{2}, 2)\).
7. [5 points (bonus)] Let \( f(x) = 1 - x^2 \). Compute

\[
\lim_{h \to 0} \frac{f \left( \frac{1}{x+h} \right) - f \left( \frac{1}{x} \right)}{h}.
\]