**Read This First!**

- Keep cell phones off and out of sight.
- Do not talk during the exam.
- You are allowed one page of notes, front and back. No other books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Please read each question carefully. Show **ALL** work clearly in the space provided. There is an extra page at the back for additional scratchwork.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.

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**Grading - For Instructor Use Only**

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1. [18 points] Evaluate the following limits. Answer either as a specific value, $+\infty$, $-\infty$, or "DNE."

(a) \[ \lim_{x \to 4} \frac{x^2 - x - 12}{x^2 - 3x - 4} = \lim_{x \to 4} \frac{(x-4)(x+3)}{(x-4)(x+1)} \]
\[ = \frac{4+3}{4+1} = \frac{7}{5} \]

(b) \[ \lim_{x \to 3} \frac{x - 5}{x - 3} \]
\[ \text{as } x \to 3^-, \quad x < 0 \]
\[ = \frac{3-5}{0^-} = \frac{-2}{0^-} = +\infty \]

(c) \[ \lim_{x \to -1} \frac{x^2 - 1}{\sqrt{x + 5} - 2} \cdot \frac{\sqrt{x + 5} + 2}{\sqrt{x + 5} + 2} = \lim_{x \to -1} \frac{(x+1)(x-1)(\sqrt{x+5} + 2)}{(x+5) - 4} \]
\[ = \lim_{x \to -1} \frac{(x+1)(x-1)(\sqrt{x+5} + 2)}{x+1} = (-2) \cdot (\sqrt{-4} + 2) \]
\[ = -8 \]
(d) \[ \lim_{x \to 2} \frac{x^2 + 2x - 7}{x^2 - 9} = \frac{4+4-7}{4-9} = \frac{1}{-5} = -\frac{1}{5} \]

(e) \[ \lim_{x \to 1} \frac{x^2 - 3x + 2}{|x-1|} \]

Consider the one-sided limits in order to simplify further:

\[ \lim_{x \to 1^-} \frac{x^2 - 3x + 2}{|x-1|} = \lim_{x \to 1^-} \frac{(x-1)(x-2)}{|x-1|} = \frac{-1}{-1} = 1 \]

\[ \lim_{x \to 1^+} \frac{x^2 - 3x + 2}{|x-1|} = \lim_{x \to 1^+} \frac{(x-1)(x-2)}{|x-1|} = \frac{-1}{1} = -1 \]

These disagree, so the overall limit \( \text{DNE} \)

(f) \[ \lim_{x \to 1^+} \frac{x^2 + 4x + 3}{x^2 - 4x + 3} \]

DPS gives \( \frac{8}{0} \); do sign analysis.

\[ \lim_{x \to 1^+} \frac{(x+1)(x+3)}{(x-1)(x-3)} = \frac{2,4}{0^+ \cdot (-2)} = -\infty \]
2. [16 points] Consider the following piecewise function.

\[
f(x) = \begin{cases} 
4 & x \leq -2 \\
4 - x^2 & -2 < x \leq 0 \\
1 + \frac{1}{x} & 0 < x < 1 \\
3 - x & x \geq 1 
\end{cases}
\]

(a) Determine all discontinuities of this function.

The individual pieces are all continuous, except \(1 + \frac{1}{x}\), which is discontinuous at \(x = 0\), but this is outside the interval where it is used.

So we need only check the break-points: \(x = -2, 0, 1\).

\(x = -2\)

LHL: \(\lim_{x \to (-2)^-} f(x) = \lim_{x \to -2^-} (4) = 4\)

RHL: \(\lim_{x \to (-2)^+} f(x) = \lim_{x \to -2^+} (4 - x^2) = 4 - (-2)^2 = 0\)

Then differ, so there's a \underline{jump discontinuity} at \(x = -2\).

\(x = 0\)

LHL: \(\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (4 - x^2) = 4 - 0^2 = 4\)

RHL: \(\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(1 + \frac{1}{x}\right) = 1 + \frac{1}{0^+} = +\infty\)

One of these is infinite, so there's an \underline{infinite discontinuity} at \(x = 0\).

\(x = 1\)

LHL: \(\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left(1 + \frac{1}{x}\right) = 1 + \frac{1}{1^-} = 2\)

RHL: \(\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3 - x) = 3 - 1 = 2\)

then agree & equal \(f(1) = 3 - 1 = 2\), so \(f(x)\) is \underline{continuous here}.

Discontinuities at \(x = -2, 0\).

(continued on reverse)
The definition of $f(x)$ is reproduced below for convenience.

$$f(x) = \begin{cases} 
4 & x \leq -2 \\
4 - x^2 & -2 < x \leq 0 \\
1 + \frac{1}{x} & 0 < x < 1 \\
3 - x & x \geq 1 
\end{cases}$$

(b) Sketch the graph of this function on the axes below.
3. [5 points] Find an equation for a line that is parallel to the line $3x + 4y = 24$ and passes through the point $(1, 2)$.

$$3x + 4y = 24$$

$$\iff 4y = -3x + 24$$

$$\iff y = -\frac{3}{4}x + 6$$

which has slope $-3/4$.

Parallel lines all have slope $-3/4$. So the desired line, in point-slope form, is

$$(y-2) = -\frac{3}{4}(x-1)$$

ie. $y-2 = -\frac{3}{4}x + \frac{3}{4}$

$$\iff y = -\frac{3}{4}x + \frac{11}{4}$$
4. [14 points] Shown below is the graph of a function $f(x)$. Determine the following limits of function values. If a value is $+\infty$ or $-\infty$, state this. If a value does not exist, answer “DNE.”

(a) $\lim_{x \to (-5)^-} f(x)$
(b) $\lim_{x \to (-5)^+} f(x)$
(c) $f(-5)$
(d) $\lim_{x \to (-2)} f(x)$
(e) $f(-2)$
(f) $\lim_{x \to 3^-} f(x)$
(g) $\lim_{x \to 3^+} f(x)$
(h) Identify all values of $x$ where $f(x)$ is discontinuous.

- $x = -5$ (jump)
- $x = -2$ (removable)
- $x = 3$ (infinite)
5. [7 points] Define \( f(x) \) and \( g(x) \) as follows.

\[
 f(x) = \frac{2 + x}{1 - x} \\
g(x) = \frac{3x + 1}{x - 2}
\]

Compute and simplify \( f(g(x)) \) as much as possible.

\[
f(g(x)) = \frac{2 + g(x)}{1 - g(x)} = \frac{2 + \frac{3x + 1}{x - 2}}{1 - \frac{3x + 1}{x - 2}} = \frac{2(x-2) + (3x+1)}{(x-2) - (3x+1)} = \frac{2x-4+3x+1}{x-2-3x-1} = \frac{5x-3}{-2x-3} = \boxed{-\frac{5x-3}{2x+3}}
\]
6. [3 points (bonus)] Let \( f(x) = 2 - \sqrt{3x+1} \). Compute
\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.
\]
Note that the answer will be in terms of \( x \).

\[
\lim_{h \to 0} \frac{(2 - \sqrt{3(x+h)+1}) - (2 - \sqrt{3x+1})}{h}
\]

\[
= \lim_{h \to 0} \frac{2 - \sqrt{3x+3h+1} - 2 + \sqrt{3x+1}}{h}
\]

\[
= \lim_{h \to 0} \frac{\sqrt{3x+1} - \sqrt{3x+3h+1}}{h} \cdot \frac{\sqrt{3x+1} + \sqrt{3x+3h+1}}{\sqrt{3x+1} + \sqrt{3x+3h+1}}
\]

\[
= \lim_{h \to 0} \frac{(3x+1) - (3x+3h+1)}{h \cdot (\sqrt{3x+1} + \sqrt{3x+3h+1})}
\]

\[
= \lim_{h \to 0} \frac{-3h}{h \cdot (\sqrt{3x+1} + \sqrt{3x+3h+1})}
\]

\[
= \frac{-3}{(\sqrt{3x+1} + \sqrt{3x+1})}
\]

\[
= -\frac{3}{2\sqrt{3x+1}}
\]