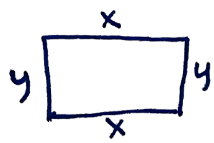
Worksheet 9 answer key

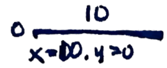
① Picture/setup

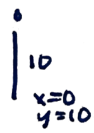


$2x + 2y = 20$ (perimeter)

want max. xy (area)

Extreme cases:





Solve

$2y = 20 - 2x$

$y = 10 - x$

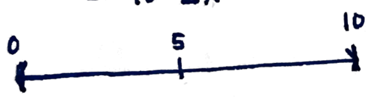
$A(x) = x(10-x)$

want max. on $[0,10]$

maximize on [0,10]

$A'(x) = 1\cdot(10-x) + x\cdot(-1)$

$= 10 - 2x$ (zero @ $x=5$)



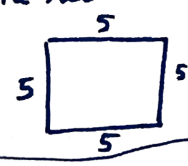
	(0, 5)	(5, 10)
$10-2x$	+	−
A	↗	↘

1st deriv. test for abs. max

$\Rightarrow$ max. on $[0,10]$ occurs at $x=5$

$\Rightarrow y = 10 - 5 = 5$

So the rect. w/ max area is a 5×5 square: (area = 25).



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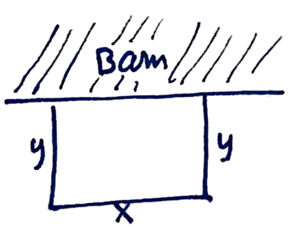


area $= xy = 50$

perim. $= x + 2y$ (want minimum)

Solve

$y = 50/x$

$f(x) = x + 2\cdot\frac{50}{x} = x + \frac{100}{x}$

want minimum on $(0,\infty)$ (any positive x is possible)

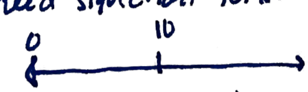
minimize on $(0,\infty)$

$f'(x) = 1 - \frac{100}{x^2}$

$= \frac{x^2 - 100}{x^2}$

$= \frac{(x+10)(x-10)}{x^2}$

only need sign chart for $x>0$:

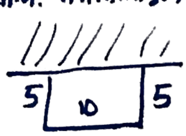


	(0, 10)	(10, ∞)
$(x+10)$	+	+
$(x-10)$	−	+
$\frac{1}{x^2}$	+	+
f'	−	+
f	↘	↗

1st DT for abs. min:

min. on $(0,\infty)$ occurs @ $x=10$, $y = \frac{50}{10} = 5$

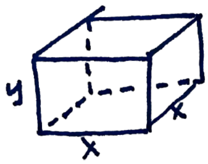
These dims. minimize fencing:



(min. fence = 20 m)

③

Picture/setup



Vol. $= x^2y = 16$

cardboard needed

$= 2\cdot(2x^2) + 4\cdot xy$ (top & bottom; 4 sides)

$= 4x^2 + 4xy$

Solve

$y = 16/x^2$

$f(x) = 4x^2 + 4x\cdot\frac{16}{x^2}$

$= 4x^2 + \frac{64}{x}$

want min on $(0,\infty)$

minimize

$f'(x) = 8x - \frac{64}{x^2}$

$= \frac{8x^3-64}{x^2}$

$= 8\cdot\frac{x^3-8}{x^2}$

(zero @ $x=2$, undef. @ $x=0$)

sign. anal. on $(0,\infty)$ only:

	$(0,2)$	$(2,\infty)$
$\frac{8}{x^2}$	+	+
x^3-8	−	+
f'	−	+
f	↘	↗

1st DT for abs. min:

abs. min. on $(0,\infty)$ @ $x=2$,

$y=4$ $(16/2^2)$

Dimensions 2×2×4 minimize the amount of cardboard

(min. is 48)

④

Setup

$xy = 18$

minimize

$x+2y$

Solve

$y = 18/x$

$f(x) = x + 2\cdot\frac{18}{x}$

$= x + 36/x$

want min. on $(0,\infty)$

$f'(x) = 1 - 36/x^2 = \frac{x^2-36}{x^2} = \frac{(x+6)(x-6)}{x^2}$ (crit. pts. ± 6 & 0)

sign anal. on $(0,\infty)$ only:

	$(0,6)$	$(6,\infty)$
$x+6$	+	+
$x-6$	−	+
$\frac{1}{x^2}$	+	+
f'	−	+
f	↘	↗

1st DT for abs. min:

abs. min on $(0,\infty)$ occurs @ $x=6$

$y=3$ $(18/6)$

So the two numbers are 3 & 6.

(5)

a) (0/0) (DSP)

$$\lim_{x\to -2} \frac{x^2+3x+2}{x^2+x-2} = \lim_{x\to -2} \frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-1)} = \frac{-2+1}{-2-1} = \frac{-1}{-3} = \boxed{1/3}$$

b) (0/0)

$$\lim_{x\to 5} \frac{25-x^2}{\sqrt{x+4}-3} = \lim_{x\to 5} \frac{(5+x)(5-x)\cdot(\sqrt{x+4}+3)}{(\sqrt{x+4}-3)(\sqrt{x+4}+3)}$$

$$= \lim_{x\to 5} \frac{(5+x)(5-x)(\sqrt{x+4}+3)}{x+4-9} = \lim_{x\to 5} \frac{(5+x)(-1)\cancel{(x-5)}(\sqrt{x+4}+3)}{\cancel{x-5}}$$

$$\overset{DSP}{=} (5+5)\cdot(-1)\cdot(\sqrt{9}+3) = 10\cdot(-1)\cdot 6 = \boxed{-60}$$

c) $|7-x| = -(7-x)$ when $x>7$

$$\lim_{x\to 7^+} \frac{|7-x|}{x^2-x-42} = \lim_{x\to 7^+} \frac{-(7-x)}{(x-7)(x+6)}$$

$$= \lim_{x\to 7^+} \frac{\cancel{x-7}}{\cancel{(x-7)}(x+6)} \overset{DSP}{=} \frac{1}{7+6} = \boxed{1/13}$$

d)

$$\lim_{x\to -5} \frac{\frac{5}{x} - \frac{1}{x+4}}{x+5} = \lim_{x\to -5} \frac{\frac{5(x+4)-x}{x(x+4)}}{x+5}$$

$$= \lim_{x\to -5} \frac{4x+20}{x(x+4)(x+5)} = \lim_{x\to -5} \frac{4\cancel{(x+5)}}{x(x+4)\cancel{(x+5)}}$$

$$\overset{DSP}{=} \frac{4}{(-5)(-5+4)} = \frac{4}{(-5)(-1)} = \boxed{4/5}$$

⑥

a) $$\frac{d}{dx}\left[\frac{\sqrt{x^3-x^{-8}}}{(x^2+5)^4}\right] = \frac{\left(\frac{d}{dx}\sqrt{x^3-x^{-8}}\right)(x^2+5)^4 - \sqrt{x^3-x^{-8}}\cdot\frac{d}{dx}(x^2+5)^4}{\left[(x^2+5)^4\right]^2}$$

$$= \frac{\frac{1}{2\sqrt{x^3-x^{-8}}}(3x^2+8x^{-9})(x^2+5)^4 - \sqrt{x^3-x^{-8}}\cdot 4(x^2+5)^3\cdot 2x}{(x^2+5)^8}$$

(don't need to simplify!)

b) $$\frac{d}{dx}\left[\left(\frac{1}{x}-\frac{1}{x^4}\right)\sqrt{x^2+1}\right] = \left(-\frac{1}{x^2}+\frac{4}{x^5}\right)\sqrt{x^2+1} + \left(\frac{1}{x}-\frac{1}{x^4}\right)\cdot\frac{1}{2\sqrt{x^2+1}}\cdot 2x$$

(don't need to simplify!)

7

$f(x) = \frac{x}{x+2}$

limit def:

$$f'(x) = \lim_{h\to 0} \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h} = \lim_{h\to 0} \frac{\frac{(x+h)\cdot(x+2) - x\cdot(x+h+2)}{(x+h+2)(x+2)}}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2x+xh+2h-x^2-xh-2x}{(x+h+2)(x+2)h}$$

$$= \lim_{h\to 0} \frac{2h}{(x+h+2)(x+2)h}$$

$$\overset{DSP}{=} \frac{2}{(x+2)(x+2)} = \boxed{\frac{2}{(x+2)^2}}$$

check, w/ quot. rule

$$f'(x) = \frac{1\cdot(x+2) - x\cdot 1}{(x+2)^2} = \frac{x+2-x}{(x+2)^2}$$

$$= \frac{2}{(x+2)^2} \quad ✓$$